Topics for Final Exam , MATH309-Spring 2013

Please read carefully all items below. Then make the review topic by topic. Note that the exam will cover most of the topics below. Do not put yourself in the situation that you come to the test without reviewing some of the topics. If you struggle with a topic, do not hesitate to come to my office hours or send me an email with your concerns. I will give special office hours for the final on Wednesday, May 1, 11 a.m-1 p.m., Thursday, May 2, 10 a.m.-12 p.m., Friday , May 3 10 a.m.-1 p.m.

The final test is on Monday, May 6 10:30 a.m.-12:30 p.m in our regular class room. It will be comprehensive, with roughly 25% on material covered since the third exam and 75% of material covered in the lectures before. Below is the list of topics that you should review in prder to be prepared to this exam:

1. **Section 3.5** To know the notion of the coordinate vector of a vector with respect to a given basis, the notion of the transition matrix from one ordered basis to another. To know how to find the transition matrix between two given bases in $\mathbb{R}^n$. To know how given a transition matrix between two bases in a vector space V and the coordinates (the coordinate vector) of a vector with respect to one basis to find the coordinates of the same vector with respect to another basis.
   Solving Problem 11, p. 165 of chapter 3 test B will be very useful here

2. **Sections 4.2-4.3**

   - To know the notion of the matrix representation of a linear transformation $L : V \to W$ with respect to a given basis in V and a given basis W (as stated in Theorem 4.2.2, page 179) and how to determine this matrix representation. To know how given the standard matrix representation of a linear transformation L from $\mathbb{R}^n$ and $\mathbb{R}^m$ to find the matrix representation of L with respect to a given bases in $\mathbb{R}^n$ and $\mathbb{R}^m$. More generally, given the matrix representation A of a linear transformation $L : V \to W$ with respect to a basis E in V and F in W to know how to find the matrix representation of L with respect other bases $\widetilde{E}$ in V and $\widetilde{F}$ in W if the transition matrices from $\widetilde{E}$ to E and from $\widetilde{F}$ and F are given.
   - To know how to implement the tasks of the previous item in the particular case when $W = V$ (and $E = F$, $\widetilde{E} = \widetilde{F}$). Namely, to know the notion of the matrix representation of a linear operator $L : V \to V$ with respect to a given basis in V and how to determine this matrix representation. To know how given the standard matrix representation of a linear operator on $\mathbb{R}^n$ to find the matrix representation of the transformation with respect to a given basis in $\mathbb{R}^n$. More generally, given the matrix representation A of a linear operator $L : V \to V$ with respect to a basis E in V to know how to find the matrix representation of L with respect another basis $\widetilde{E}$ if the transition matrices from $\widetilde{E}$ to E is given. To know the notion of similar matrices.

   Solving Problems 2, 8, 9, 10, p. 197 of chapter 4 test B will be very useful here

3. **Section 6.1-6.3**

   - To know the notion of eigenvalues, eigenvectors and eigenspaces of matrices/linear operators, how to find the eigenvalues as the roots of characteristic polynomial and the relation of eigenvalues of a matrix to the determinant and the trace; to know the properties of eigenvalues of similar matrices (Theorem 6.1.1, page 293);

- To know the notion of the algebraic and geometric multiplicity of an eigenvalue (will be given in class of Friday, March 22), diagonalizable and defective matrices/operators and how to determine whether the matrix is diagonizable;
- In the case when a matrix A is diagonizable to know how to find the diagonal matrix D which is similar to A and a matrix X which diagonalizes A (in other words, how to factor A into the product XDX^{-1}, where D is diagonal by writing explicitly what is D and X)
- To know the notion of the exponential of a matrix. Using the representation $A = XDX^{-1}$ of the previous item to know how to find A^k for any k and how to find e^{At}. To know how to use the latter in order to find the solution of initial value problems for systems of ordinary differential equation: $\mathbf{x}' = A\mathbf{x},\ \mathbf{x}(0) = \mathbf{x}_0$

Solving Problems 1, 2, 3, 4, 7 p. 384-385 of chapter 6 test B will be very useful here

4. **Section 5.5 *Orthonormal sets.*** To know to determine whether a given set of vectors is orthogonal or orthonormal. To know how to find the coordinates of a given vector with respect to an orthonormal basis (Theorem 5.5.2 page 243) and to know how to find the inner product and the length of the vector via its coordinates with respect to the orthomormal basis (Corollaries 5.5.3 and 5.5.4 in the same page). To know how given a point v in an inner product space to find the closest point to this point in a given subspace if an orthonormal basis is given in this subspace (as in the problems 29 and 30 of the homework assignment #10, this closest point is called the best least square approximation, reviewing these problems and solving problem 12, p.281 from the Chapter 5 test B will be very helpful here)

5. To know how to find the Fourier series of a periodic function. To know the Dirichlet condition and to what function the Fourier series of a function satisfying Dirichlet condition converges (especially what happens at the points of discontinuity of the original function). To know the Parseval identity for the coefficients of the Fourier series (lecture 4, class of April 17)
reviewing problem 1 of homework 11 might be very helpful here

6. To know how to find the half range Fourier sine and cosine series of a function, to know their relation to odd and even functions, to know to to what function the half range Fourier series converges (lecture 5, class of April 19).
reviewing problem 2 and 3 (a) and (b) of homework 11 might be very helpful here

7. To know how to apply the method of separation of variables and half-range Fourier series to the one-dimensional wave equation with fix ends (lecture 7 of class of April 26, the third column of the table on pages 1, more detailed treatment of such equation can be found in my webpage starting from May 1, solving problem 1 of homework 12 might be very helpful here).

8. To know how to apply the method of separation of variables and the Fourier series to the Laplace equation on the disk via the passage to the polar coordinates (lecture 7 of class of April 26, solving problem 2 of homework 12 might be very helpful here).

Also it is highly recommended to review all problems from homework assignments and the examples given during the class on the topics listed above.