Homework Assignment 1 in MATH309-Spring 2013, ©Igor Zelenko

due January 23, 2013

Please read carefully what you are supposed to find in each problem.

Sections covered: 1.1, 1.2: consistency/inconsistency, back substitution, augmented matrix of the system, row echelon form and Gaussian elimination, reduced row echelon form and Jordan-Gauss reduction.

1. In each of the following systems , interpret each equation as a line in the plane. For each system graph the line, decide whether the system is consistent or not and interpret geometrically the number of solutions. Then, if the system is consistent, find all solutions algebraically (you can use any method here).

   (a) $\begin{aligned} 2x_1 + x_2 &= 6 \\ x_1 - 3x_2 &= -4 \end{aligned}$ (b) $\begin{aligned} 6x_1 - 3x_2 &= -4 \\ -8x_1 + 4x_2 &= 3 \end{aligned}$ (c) $\begin{aligned} 6x_1 - 3x_2 &= -3 \\ -8x_1 + 4x_2 &= 4 \end{aligned}$

2. Use the back substitution to solve the following system of equations:

$$\begin{aligned} 3x_1 &+ 5x_2 &+ 2x_3 &- 4x_4 &+ x_5 &= 4 \\ &2x_2 &+ x_3 &- 2x_4 &+ 4x_5 &= -4 \\ & &3x_3 &+ 5x_4 &- x_5 &= 0 \\ & & &10x_4 &+ 5x_5 &= 5 \\ & & & &7x_5 &= -7 \end{aligned}$$

3. Section 1.2, p.23, Problem 1 (if your answer is no, explain what condition of the corresponding definition fails)

4. The augmented matrices are in row echelon form. For each case, indicate whether the corresponding system is consistent. If the system has a unique solution, find it. If the system has infinite many solution indicate which variables can be taken as free.

   (a) $\left(\begin{array}{cc|c} 1 & 5 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{array} \right)$ (b) $\left(\begin{array}{ccc|c} 1 & 5 & 3 & -6 \\ 0 & 1 & 7 & 5 \\ 0 & 0 & 1 & 1 \end{array} \right)$ (c) $\left(\begin{array}{cccc|c} 1 & 5 & -2 & 3 & 5 \\ 0 & 0 & 1 & 4 & -3 \end{array} \right)$

5. For each of the systems of equations that follow, write the corresponding augmented matrix, then transform it to a row echelon form, then indicated whether the system is consistent. If the system is consistent, use either back substitution or transform it to the reduced row echelon form and find all solutions.

   (a) $\begin{cases} 2x_1 + 3x_2 - x_3 + x_4 &= 2 \\ x_1 + 2x_2 + x_3 - x_4 &= 1 \\ 3x_1 + 5x_2 &= 3 \\ x_1 + x_2 - 2x_3 + 2x_4 &= 1 \end{cases}$.

   (b) $\begin{cases} x_1 - 3x_2 + 2x_3 + x_4 &= 1 \\ -3x_1 + 10x_2 - 5x_3 - x_4 &= -1 \\ -2x_1 + 7x_2 - 2x_3 - x_4 &= 5 \end{cases}$;

   (c) $\begin{cases} x_1 + 2x_2 - x_3 = 6 \\ -x_1 + 2x_2 + 2x_3 = 5 \\ 4x_2 + x_3 = 10 \end{cases}$

6. **(bonus-15 points)** Consider a linear system whose augmented matrix is in the form: $\left(\begin{array}{ccc|c} 1 & 5 & 4 & 6 \\ 2 & 9 & 1 & 2 \\ 1 & 2 & a & b \end{array} \right)$

   (a) For what values a and b will the system have infinite many solutions?

   (b) For what values a and b will the system be inconsistent?