

Homework Assignment 10 in MATH309-Spring 2013, ©Igor Zelenko

due April 19, 2013 . Show your work in all exercises

Topics covered: Orthonormal sets (section 5.5), boundary value problems for PDEs, classification of second order linear PDE's, separability and beginning of method of separation of variables

Section 5.5

1. Exercise 2, page 257.
2. Solve any 2 out of the following 3 problems: exercises 5, 6, 7 page 257. If you solve all 3 of them you get **bonus of 10 points**.
3. Exercise 9, pages 257 – 258.
4. Solve one of the following 2 problems: exercises 29, 30, pages 258 – 259. If you solve both of them you get **bonus of 12 points**. In the item (c) there least square approximation is the linear function which is closest to the given function with respect to the given norm.

Exercises related to PDEs

5. For what value(s) of a will $u(x, t) = f(x + at)$ solve the PDE $u_{xx} + 2u_{xt} - 3u_{tt} = 0$ for any twice differentiable function f of single variable.
6. Find where the following PDE's are hyperbolic, elliptic, or parabolic (note that the same PDE may belong to different classes at different points):
 - (a) $u_{tt} = tu_x$
 - (b) $u_{tt} + 4u_{xt} + 4u_{xx} + x^2u_x - t^2u_t + \sin(tx)u = 0$;
 - (c) $u_{xx} + xu_{xy} + yu_{yy} = 0$.
7. Do the variables in the following PDE's separate? If so, write (but do not solve) the resulting ODE's:
 - (a) $u_{xy} = u$;
 - (b) $u_{tt} + 2u_t - 4u_{xx} + u = 0$
 - (c) $(x^2 + t^2)u_{tt} = u_{xx}$
 - (d) $u_{xx} + u_{yy} = u_t$ (Note that here we have 3 variables).
8. Solve $u_t = u_{xx}$ on $0 < x < 3, t > 0$ with the initial and boundary conditions

$$u(x, 0) = 3 \cos\left(\frac{\pi x}{3}\right) + 2 \cos(\pi x), \quad u_x(0, t) = u_x(3, t) = 0 \text{ for all } t > 0 \text{ (insulated ends).}$$