Bonus Homework Assignment 12 in MATH309-Spring 2013, ©Igor Zelenko

Submit it on Friday, May 3 2013 between 10 a.m.-13 p.m. to my office, Milner 324, or slide it under my office door but not later than on the same day, May 3 2013 . Show your work in all exercises.

Topics covered: Applications of Fourier series to the one dimensional wave equation and Laplace equation on a disk; applications of Bessel functions of the first kind to the heat transfer on the circular plate and the vibration of a drum.

1. Solve the following one-dimensional wave equation

$$u_{tt} = 4u_{xx}, \quad 0 < x < 3, \quad t > 0$$

with the initial conditions

$$u(x,0) = x^2, \quad u_t(x,0) = 0$$

and the boundary conditions

$$u(0,t) = u(3,t) = 0.$$

(Hint: Here you can use the scheme of the third column of the table on page 1 of Lecture Notes of Friday, April 26. You can find more detailed treatment of such kind of problems in the supplementary notes on my webpage, starting from May 1)

2. Solve the following boundary value problem:

$$u_{xx} + u_{yy} = 0 \text{ for } x^2 + y^2 < 1,$$

$$u(x,y) = y^3 \text{ for } x^2 + y^2 = 1.$$

(Hint: pass to the polar coordinates and use the scheme given in the lecture 7 of Friday, April 26)

The material needed for the next problem will be not included in the final test but it will be still counted for the grade of this homework assignment. In order to do this problem I recommend you to review the lecture notes of classes of April 29/30 that you can find on my web-page starting from May 1. These lecture notes contain more material than we covered in class

3. In both items of this problem $J_n(r)$ denotes the Bessel function of the first kind of order n and λ_{ni} denotes the ith positive zeros of J_n. You should express your solution in terms of the appropriate J_n and λ_{ni}. Also you can use the following formula (see the lecture notes for the explanation):

$$\int_0^1 rJ_n(\lambda r)\,dr = \frac{J_{n+1}^2(\lambda)}{2}.$$

(a) (*the heat equation for the circular plate*) A circular plate of radius 1 and diffusivity $k = 3$ has initial temperature (in polar coordinates)

$$u(r,\theta,0) = -3J_0(\lambda_{02}r) + 4J_2(\lambda_{23}r)\cos 2\theta$$

and the rim is kept in temperature zero. Find the temperature at time t, assuming that thefaces are insulated so that it is strictly problem in the plane, i.e. the heat is transferred in the plane.

(b) (*the vibration of the drum*) Consider a drum with a circular head of radius 1. One pushes the head a little so that its intial shape (in polar coordinates) is

$$u(r,\theta,0) = \frac{1}{10}J_1(\lambda_{12}r)\sin\theta - \frac{1}{5}J_3(\lambda_{35}r)\cos 3\theta$$

and then let it go (starting with zero as initial velocity). Find the shape of the head at any time $t > 0$.