Homework Assignment 3 in MATH309-Spring 2013, ©Igor Zelenko

due February 6, 2013 (you may earn 30 points of bonus). **Show your work in all exercises.**

Topics covered: Finding inverse both via Jordan-Gauss reduction (as in section 1.5) and via the adjoint matrix (section 2.3); Determinants (Sections 2.1-2.3 and my lecture notes on permutations): definition of determinant both using expansion along a row or a column and using the permutations, properties of the determinant, the characterization of non-singular matrices via determinant, Cramer's rule

1. Calculate the determinants of the following matrices (show your work):

(a) $\begin{pmatrix} 0 & a & 0 \\ b & c & d \\ 0 & e & 0 \end{pmatrix}$; **(b)** $\begin{pmatrix} -10 & 4 & -1 & 6 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 7 & -3 & 2 & 8 \end{pmatrix}$ **(c)** $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$

Hint: Choose expansions along appropriate rows.

Hint: Use row operations.

2. In each item find the inverse of the matrix if it exists using the method indicated there (show all of your work):

   (a) $\begin{pmatrix} 1 & 2 & -1 \\ 3 & 7 & -4 \\ 7 & 6 & -5 \end{pmatrix}$ using the Jordan-Gauss reduction;

   (b) $\begin{pmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{pmatrix}$ using the adjoint matrix formula of section 2.3, page 99.

   (c) $\begin{pmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ 7 & 16 & -21 \end{pmatrix}$ using any method you want.

3. Let A be a 6×6 matrix. Determine whether the following term appears in the expansion of the determinant and if yes with what sign (justify your answer):

   (a) $a_{11}a_{22}a_{33}a_{44}a_{55}a_{66}$; **(b)** $a_{12}a_{23}a_{35}a_{46}a_{52}a_{64}$; **(c)** $a_{13}a_{24}a_{36}a_{42}a_{55}a_{61}$ **(d)** $a_{16}a_{25}a_{34}a_{43}a_{52}a_{61}$.

4. Let A and B be 7×7 matrices such that $\det(A) = 5$ and $\det(B) = 7$. Find the values of

   (a) $\det(3A^2B)$; **(b)** $\det(4A^{-1}B^2)$ **(c)** $\det(AB^{-2})$ (here $B^{-2} := (B^{-1})^2$).

5. Use Cramer's rule to solve the following system of linear equations if possible

$$\begin{aligned} 4x_1 - x_2 - x_3 &= 1 \\ 2x_1 + 2x_2 + 3x_3 &= 10 \\ 5x_1 - 2x_2 - 2x_3 &= -1 \end{aligned}$$

6. (a) Let A be an $n \times n$ matrix and α be a scalar. Show that $\det(\alpha A) = \alpha^n \det(A)$;

   (b) Let A be a 3×3 skew-symmetric matrix, i.e. $A^T = -A$. Find $\det A$ (justify your answer);

   (c) (**bonus 10 points**) What can you say about $det(A)$ if A is an $n \times n$ skew-symmetric matrix with arbitrary odd n. Justify your answer (*Hint: Use item (a) and the fact that* $\det(A^T) = \det(A)$).

7. (**bonus 10 points**) Show that $\begin{vmatrix} a & 1 & 1 & 1 \\ 1 & a & 1 & 1 \\ 1 & 1 & a & 1 \\ 1 & 1 & 1 & a \end{vmatrix} = (a+3)(a-1)^3$

8. (**bonus 10 points**) Section 2.2, p. 97, problem 12.