

Problem 5 HWK #3 corrected solution

$$\begin{cases} 4x_1 - x_2 - x_3 = 1 \\ 2x_1 + 2x_2 + 3x_3 = 10 \\ 5x_1 - 2x_2 - 2x_3 = -1 \end{cases} \Rightarrow A = \begin{pmatrix} 4 & -1 & -1 \\ 2 & 2 & 3 \\ 5 & -2 & -2 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix}$$

$$\det A = \begin{vmatrix} 4 & -1 & -1 \\ 2 & 2 & 3 \\ 5 & -2 & -2 \end{vmatrix} = 4 \begin{vmatrix} 2 & 3 \\ -2 & -2 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} - \begin{vmatrix} 2 & 2 \\ 5 & -2 \end{vmatrix} =$$

$$= 4(-4+6) + (-4-15) - (-4-10) = 8 - 19 + 14 = 3$$

By Cramer's rule

$$x_1 = \frac{\begin{vmatrix} 1 & -1 & -1 \\ 10 & 2 & 3 \\ -1 & -2 & -2 \end{vmatrix}}{3} = \frac{(-4+6) + (-20+3) - (-20+2)}{3} = \frac{2-17+18}{3} = \frac{3}{3} = 1$$

$$x_2 = \frac{\begin{vmatrix} 4 & -1 & -1 \\ 2 & 10 & 3 \\ 5 & -1 & -2 \end{vmatrix}}{3} = \frac{4(-20+3) - (-4-15) - (-2-50)}{3} =$$
$$= \frac{-68 + 19 + 52}{3} = \frac{19-16}{3} = 1$$

$$x_3 = \frac{\begin{vmatrix} 4 & -1 & 1 \\ 2 & 2 & 10 \\ 5 & -2 & -1 \end{vmatrix}}{3} = \frac{4(-2+20) + (2-50) + (-1+10)}{3} = \frac{72-52-14}{3} = \frac{6}{3} = 2$$

$\Rightarrow (1, 1, 2)$