Homework Assignment 5 in MATH309-Spring 2013, ©Igor Zelenko

due February 25, 2013 . Show your work in all exercises.

Sections covered 3.3, 3.4, 3.5: Linearly dependence/independence, basis and dimension, change of basis

1. Determine whether the given vectors are linear independent in a given vector spaces. If not, pare down them to form a basis of their span (in other words, choose a subset of them that constitute a basis of their span) and find the dimension of this span (justify your answers):

   (a) $\begin{pmatrix}3\\2\\1\end{pmatrix}, \begin{pmatrix}1\\3\\2\end{pmatrix}, \begin{pmatrix}4\\5\\2\end{pmatrix}$ in $\mathbb{R}^3$;

   (b) $\begin{pmatrix}3\\2\\1\end{pmatrix}, \begin{pmatrix}1\\3\\2\end{pmatrix}, \begin{pmatrix}6\\11\\7\end{pmatrix}, \begin{pmatrix}1\\10\\3\end{pmatrix}$ in $\mathbb{R}^3$;

   (c) $\begin{pmatrix}1\\3\\0\\5\end{pmatrix}, \begin{pmatrix}4\\0\\2\\1\end{pmatrix}, \begin{pmatrix}6\\6\\2\\11\end{pmatrix}, \begin{pmatrix}-7\\3\\-4\\3\end{pmatrix}$ in $\mathbb{R}^4$;

   (d) $\begin{pmatrix}3 & -2\\4 & 1\end{pmatrix}, \begin{pmatrix}-1 & 3\\2 & 5\end{pmatrix}, \begin{pmatrix}11 & -12\\8 & -7\end{pmatrix}, \begin{pmatrix}2 & 1\\6 & 4\end{pmatrix}$ in $\mathbb{R}^{2\times 2}$;

   (e) x^2+3x-4, $2x^2-5$, $2x-1$, 2 in P_3;

   (f) $e^{r_1 x}$, $e^{r_2 x}$, $e^{r_3 x}$ in $C[-1,1]$, where r_1 , r_2 , and r_3 are pairwise distinct;

   (g) 1, $\cos 2x$, $\cos^2 x$ in $C[-\pi,\pi]$.

2. Exercise 6, page 137.

3. Exercise 12, page 138.

4. Exercise 13, page 138

5. Find a basis of the given subspace S of a given vector space, if

   (a) S consists of all vectors in $\mathbb{R}^4$ of the form $(a+3b+c, -b+2c, 3a+2b-2c, c-b)^T$;

   (b) S is the subspace in P_3 consisting of all polynomials of the form $ax^2+(2b+a)x+b+2a$;

   (c) S is a subspace of P_3 consisting of all polynomials p such that $p(1)=0$;

   (d) (bonus **10 points**) S is a subspace of P_3 consisting of all polynomials p such that $p(0) = p(1) = 0$

6. Let $\mathbf{v}_1 = (1,2,4)^T$, $\mathbf{v}_2 = (-1,2,0)^T$, $\mathbf{v}_3 = (2,4,0)^T$, $\mathbf{u}_1 = (0,2,1)^T$, $\mathbf{u}_2 = (-2,1,0)^T$, $\mathbf{u}_3 = (1,1,1)^T$

   (a) Find the transition matrix from $\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3\}$ to $\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3\}$

   (b) If $\mathbf{x} = 3\mathbf{v}_1 - 4\mathbf{v}_2 + \mathbf{v}_3$, determine the coordinates of $\mathbf{x}$ with respect to $\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3\}$.