# Homework Assignment 7 in MATH309-Spring 2013, ©Igor Zelenko

**due March 20, 2013** . **Show your work in all exercises.**

*Sections covered 4.2, 4.3* **Total of 130 points**

1. Assume that a linear transformation $L : \mathbb{R}^3 \to \mathbb{R}^2$ is given by
$$L\big((x_1, x_2, x_3)^T\big) = (2x_1 - 3x_2 + x_3, -4x_1 + x_3)^T.$$
Find a matrix $A$ such that $L(\mathbf{x}) = A\mathbf{x}$ for every $\mathbf{x} \in \mathbb{R}^3$.

2. Find the standard matrix representation for each of the following linear operators (a linear operator means a linear transformation from a vector space to itself, the standard matrix representation of a linear operator in $\mathbb{R}^n$ is the matrix representation with respect to the standard basis of $\mathbb{R}^n$):
   (a) $L$ is the linear operator that rotates each $\mathbf{x}$ in $\mathbb{R}^2$ by $\frac{\pi}{3}$ in the clockwise direction;
   (b) $L$ is the linear operator that first triples the length of $\mathbf{x}$ and then reflects the obtained vector about the line $x_2 = -x_1$;
   (c) (**bonus 10 points**) $L$ is the linear operator that first rotates each $\mathbf{x}$ in $\mathbb{R}^2$ by $\frac{\pi}{3}$ in the clockwise direction, then triples the length of the obtained vector and then reflects the resulting vector about the line $x_2 = -x_1$. What matrix operation can you use in order to obtain the matrix required in this item from the matrices obtained in item (a) and (b)?

3. Let $\mathbf{b}_1 = (1, -2, -4)^T$, $\mathbf{b}_1 = (3, 2, -1)^T$, $\mathbf{b}_1 = (2, 3, -2)^T$ and $L$ be the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ defined by $L\big((x_1, x_2)^T\big) = (x_1 - 3x_2)\mathbf{b}_1 + (2x_1 + 4x_2)\mathbf{b}_2 - x_1\mathbf{b}_3$. Find the matrix $A$ representing $L$ with respect to the ordered basis $\{\mathbf{e}_1, \mathbf{e}_2\}$ and $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$.

4. Let $E = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$, $F = \{\mathbf{b}_1, \mathbf{b}_2\}$ be as in exercise 18 page 189. Let
$$L\big((x_1, x_2, x_3)^T\big) = (-x_1 + x_2 - 2x_3, x_1 - 4x_3)^T.$$
Find the matrix representing $L$ with respect to the ordered bases $E$ and $F$.

5. The linear transformation $L$ defined by
$$L(p(x)) = \frac{1}{2}p''(x) + 2p(1)$$
maps $P_4$ to $P_2$. Find the matrix representation of $L$
   (a) with respect to the ordered bases $\{1, x, x^2, x^3\}$ and $\{1, x\}$;
   (b) (**bonus 10 points**) with respect to the ordered basis $\{x^3, x^2, x, 1\}$ and $\{1 + x, 1 - x\}$.

6. Let $\{\mathbf{u}_1, \mathbf{u}_2\}$ be as in exercise 1 on page 194 and $L$ be the operator on $\mathbb{R}^2$ given by
$$L\big((x_1, x_2)\big) = (5x_1 - 3x_2, -2x_1 + 4x_2)^T.$$
Find the matrix representing $L$ with respect to to the basis $\{\mathbf{u}_1, \mathbf{u}_2\}$.

7. The linear operator on $\mathbb{R}^3$ is given by
$$L(\mathbf{x}) = (15x_1 - 11x_2 + 5x_3, 20x_1 - 15x_2 + 8x_3, 8x_1 - 7x_2 + 6x_3)^T.$$
Find the matrix of this operator in the basis $\{(2, 3, 1)^T, (3, 4, 1)^T, (1, 2, 3)^T\}$.

8. (**bonus 10 points**) Let $\{\mathbf{u}_1, \mathbf{u}_2\}$ and $\{\mathbf{v}_1, \mathbf{v}_2\}$ be the ordered bases of $\mathbb{R}^2$, where $\mathbf{u}_1 = (-2, 1)^T$, $\mathbf{u}_2 = (0, 1)^T$, $\mathbf{v}_1 = (1, -1)^T$, $\mathbf{v}_2 = (1, 1)^T$. Assume that a linear operator $L$ is represented by the matrix $B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ with respect to the basis $\{\mathbf{u}_1, \mathbf{u}_2\}$. Find the matrix $A$ representing $L$ with respect to the basis $\{\mathbf{v}_1, \mathbf{v}_2\}$.