Homework Assignment 9 in MATH309-Spring 2013, ©Igor Zelenko

due April 8, 2013 . Show your work in all exercises

Sections covered are 5.1, 5.4.

1. Let $\mathbf{x}$ and $\mathbf{y}$ be linearly independent vectors in $\mathbf{R}^n$. If $||x|| = 3$ and $||y|| = 4$, what, if anything, can we conclude about the possible values of $|\mathbf{x}^T\mathbf{y}|$?

2. Let $A = (1, -2, -3, 4)$ be a point in $\mathbb{R}^4$ and $\mathbf{y} = (-2, 3, -4, -2)^T$. Let $L = Span\{\mathbf{y}\}$. Among all points on L find the point closest to A and find the distance from the point A to the line L.

3. Find the distance from the point $(3, -1, 4, -2)$ to the hyperplane

$$2(x_1 - 2) + 3(x_2 + 1) + 4(x_3 - 5) + 5(x_4 + 4) = 0$$

   in $\mathbb{R}^4$.

4. Exercise 18, page 213.

5. In items (a)-(f) below I give two vectors $\mathbf{y}$ and $\mathbf{z}$ in a given vector space V with a given inner product. In each item answer the following question:

   (i) Find the inner product $\langle \mathbf{y}, \mathbf{z} \rangle$;

   (ii) Find the distance between $\mathbf{y}$ and $\mathbf{z}$;

   (iii) Find the angle between the vectors $\mathbf{y}$ and $\mathbf{z}$ (with respect to the given inner product in V)

   (iv) Find the vector projection of $\mathbf{y}$ onto $\mathbf{z}$.

   (a) $\mathbf{y} = (1, 3, 1, 1)^T$, $\mathbf{z} = (1, -3, 1, 1)^T$ in $V = \mathbb{R}^4$ with the standard inner product.

   (b) The vectors $\mathbf{y}$ and $\mathbf{z}$ are as in the previous item but the inner product in $\mathbb{R}^4$ is given by the formula (1) of page 232 with the weight vector $\mathbf{w} = (3, 1, 3, 3)$;

   (c) $\mathbf{y} = \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix}$, $\mathbf{z} = \begin{pmatrix} 1 & 1 \\ -3 & 1 \end{pmatrix}$, where $V = \mathbb{R}^{2\times 2}$, the space of 2×2-matrices, with the inner product given by formula (2), page 232;

   (d) $\mathbf{y} = e^{-2x}$, $\mathbf{z} = e^{2x}$ in $V = C[0, 1]$ with the inner product defined by formula (3) of page 232;

   (e) $\mathbf{y} = x^2$, $\mathbf{z} = x^3$ in $V = C[-1, 1]$ with the inner product defined by formula (3) of page 232;

   (f) $\mathbf{y} = x^2$, $\mathbf{z} = x^3$ in $V = P_5$ (the space of polynomials of degree < 5) with the inner product defined by formula (5) of page 233, where $x_i = \frac{i-3}{2}$ for $i = 1, \ldots 5$.

6. Solve one of the two exercises 18 or 26 of page 240. If you solve both you may get **8 points of bonus**.