

NAME (printed neatly) _____ QUIZ#12 GRADE _____

Directions for taking quizzes: the same as in the previous quizzes.

1. Here is a partially completed Cayley table of a certain group of order 4 and assume that it is NOT KNOWN A PRIORI THAT THIS GROUP IS ABELIAN. Fill in the rest of the table. JUSTIFY EACH STEP OF THE FILLING.

	a	b	c	d
a	b	a	d	c
b	a	b	c	d
c	d	c	b	a
d	c	d	a	b

2. Consider the cyclic group C_{18} of order 18 and let x be its generator.

- (a) List all k such that x^k is a generator of C_{18} . Justify your answer;
 (b) List all k such that x^k has order 6. Justify your answer.

1. (i) $bd = d \Rightarrow \boxed{b=e}$ (the identity) $\Rightarrow ab = ba = a, b^2 = b,$

$cb = bc = c, db = d;$

(ii) Since in the first column only d is missing $\Rightarrow ca = d$
 (iii) The last two spots in the first row must be occupied by c or d (because a and b are already there) but $ac \neq c$

(otherwise $a = e$ but we know that $b = e$; other explanation: if $ac = c$ then c will appear twice in the third column) $\Rightarrow ac = d \Rightarrow ad = c$

(iv) From the third column: $c^2 = b$;
 From the third row: $cd = a$
 From the fourth row/column: $d^2 = a$

Rem This group is actually isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$ (all non-identity elements here have order 2)

2. a) x^k is a generator $\Leftrightarrow \gcd(k, 18) = 1 \Rightarrow \boxed{k = 1, 5, 7, 11, 13, 17}$
 order of $x^k = \frac{18}{\gcd(k, 18)} = 6 \Leftrightarrow \gcd(k, 18) = 3 \Rightarrow \boxed{k = 3, 15}$

-you can continue your solution in the next page, if you do not have enough space