NAME (printed neatly)________________ QUIZ#13 GRADE ______

Directions for taking quizzes: the same as in the previous quizzes.

1. Let G_{10} be the group of invertible congruence classes modulo 10.
   (a) Write down the distinct left cosets of the group $\{[1]_{10}, [9]_{10}\}$;
   (b) Determine whether G_{10} is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$ or $\mathbb{Z}_4$.

2. Let G be a finite group of order $o(G)$.
   (a) Based on Lagrange's theorem, what are the possible orders of elements of G?
   (b) Assume that G is the group G_{15} (of invertible congruence classes modulo 15). What, according to item (a), are *possible* orders of G and which of this integers are actual orders of element of G.

1(a) $\varphi(10) = \varphi(2\cdot 5) = 1\cdot 4 = 4$

The subgroup $\{[1]_{10}, [9]_{10}\}$ has order $2 \Rightarrow$ It has $\frac{4}{2} = 2$ left cosets (which in this case ~~actually~~ equal to right cosets): one of them is $\{[1]_{10}, [9]_{10}\}$ and another one is the complement of this subgroup to the whole G_{10} i.e. $\{[3]_{10}, [7]_{10}\}$

Answer: $\boxed{\{[1]_{10}, [9]_{10}\} \text{ and } \{[3]_{10}, [7]_{10}\}}$

Remark You could also argue for the second coset as follows:

If $H = \{[1]_{10}, [9]_{10}\}$ then $[3]_{10} H = \{[3]_{10}, \underset{[7]_{10}}{\underset{\shortparallel}{[27]_{10}}}\} = \{[3]_{10}, [7]_{10}\}$

This is what you do usually if you have more than two cosets.

(b) Check orders of non-identity elements: $[3]_{10}^2 = [9]_{10} \neq [1]_{10}$, $[3]_{10}^4 = [81]_{10} = [1]_{10}$

$\Rightarrow$ there is an element of order 4 $\Rightarrow G_{10} \sim \mathbb{Z}_4$

2 (a) all divisors of G are potential orders by Lagrange's thm

(b) $o(G_{15}) = \varphi(15) = \varphi(3\cdot 5) = (3-1)(5-1) = 8 \Rightarrow$

possible orders are divisors of 8 $\Rightarrow \boxed{1, 2, 4, 8}$

-you can continue your solution in the next page, if you do not have enough space

For actual orders list the elements of G_{15} and check their orders

$G_{15} = \{[1]_{15}, [2]_{15}, [4]_{15}, [7]_{15}, [8]_{15}, [11]_{15}, [13]_{15}, [14]_{15}\}$

ord $[1]_{15} = \boxed{1}$, ord $[2]_{15} = \boxed{4}$ ($2^2 \equiv 4 \mod 15$
$2^4 \equiv 16 \equiv 1 \mod 15$

See the back page

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$\mathrm{ord}\,[4]_{15} = \boxed{2}$ $(4^2 \equiv 1 \bmod 15)$

$\mathrm{ord}\,[7]_{15} = 4$ $(7^2 \equiv 49 \equiv 4 \bmod 15 \Rightarrow 7^4 \equiv 16 \equiv 1 \bmod 15)$

$\mathrm{ord}\,[8]_{15} = 4$ $(8 \equiv -7 \bmod 15)$

$\mathrm{ord}\,[11]_{15} = 2$ $(11 \equiv -4 \bmod 15)$

$\mathrm{ord}\,[13]_{15} = 4$ $(13 \equiv -2 \bmod 15)$

$\mathrm{ord}\,[14]_{15} = 2$ $(14 \equiv -1 \bmod 15)$

The <u>actual</u> orders are $\boxed{1,2,4}$

<u>Rem</u> Actually it can be shown that $G_{15} \cong \mathbb{Z}_2 \times \mathbb{Z}_4$ and in general if $n = pq$ where p and q are distinct <u>odd</u> primes then $G_n \cong \mathbb{Z}_{p-1} \times \mathbb{Z}_{q-1}$ (there is an explicit formula also for general n in term of the prime factorization of n).