NAME (printed neatly)_____ QUIZ#14 GRADE _____

Directions for taking quizzes: the same as in the previous quizzes.

1. Find the missing digit in the following ISBN-10: 03057□6723.
2. Let $f : \mathbf{B}^2 \to \mathbf{B}^7$ be the linear code with the generator matrix

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Determine how many errors the corresponding code detects and how many errors it corrects.

1. Let d be a missing digit $\Rightarrow$

$10\times0 + 9\times3 + 8\times0 + 7\times5 + 6\times7 + 5\times d + 4\times6 + 3\times7 + 2\times2 + 3 \equiv 0 \mod 11$

0 $27\equiv5 \mod 11$ 0 $35\equiv2 \mod 11$ $42\equiv-2 \mod 11$ $24\equiv2 \mod 11$ $21\equiv-1 \mod 11$ 4

$\Rightarrow$ $5+2-2+2-1+7+5d \equiv 0 \mod 11 \Leftrightarrow 5d+13\equiv 0 \mod 11 \Leftrightarrow$

$5d \equiv -2 \mod 11$

$[5]_{11}^{-1}$?

$\left(\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 11 \end{array}\right) \xrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{cc|c} 1 & 0 & 5 \\ -2 & 1 & 1 \end{array}\right) \Rightarrow [5]_{11}^{-1} = [-2]_{11} \equiv [9]_{11}$

$[d]_{11} = [5]_{11}^{-1}[-2]_{11} = [-2]_{11}[-2]_{11} = [4]_{11} \Rightarrow \boxed{d=4}$

2. List nonzero codewords:

$10 \to 1011010 \to wt=4$
$01 \to 0101111 \to wt=5$
$11 \to 1110101 \to wt=5$

$\Rightarrow$ minimal weight of a nonzero codeword is 4

Detection of k errors $\Leftrightarrow$ $k+1 \le 4 \Rightarrow k \le 3 \Rightarrow$ detects 3 errors

Correction of k errors $\Leftarrow$ $2k+1 \le 4 \Rightarrow 2k \le 3 \Rightarrow k \le 1 \Rightarrow$ corrects 1 error

-you can continue your solution in the next page, if you do not have enough space