

NAME (printed neatly) _____ QUIZ#1 GRADE _____

Directions for taking quizzes: Write your name legibly where indicated on both sides of this paper. On the reverse side of this paper, circle the letter category which corresponds to the first letter of your last name. After you have completed this quiz, fold this paper lengthwise such that the side with your solution is in the inside of the fold (so your quiz grade will be hidden when returning papers.) Turn your quiz in on the appropriate pile as determined by the first letter of your last name. Follow the Aggie Honor Code!

- Using the Euclidean algorithm, find the greatest common factor of 297 and 1008.
- If d is your answer in the previous item, express d in the form $d = 297x + 1008y$, where x and y are integers, i.e. find these integers x and y explicitly.

1. Step 1

$$\begin{array}{r} 3 \\ 297 \overline{)1008} \\ \underline{891} \\ 117 \end{array}$$

$r_1 = 117$

Step 2

$$\begin{array}{r} 2 \\ 117 \overline{)237} \\ \underline{234} \\ 63 \end{array}$$

$r_2 = 63$

Step 3

$$\begin{array}{r} 1 \\ 63 \overline{)117} \\ \underline{63} \\ 54 \end{array}$$

$r_3 = 54$

Step 4

$$\begin{array}{r} 1 \\ 54 \overline{)63} \\ \underline{54} \\ 9 \end{array}$$

$r_4 = 9$

Step 5

$$\begin{array}{r} 6 \\ 9 \overline{)54} \\ \underline{54} \\ 0 \end{array}$$

$r_5 = 0$

30/50 if there is an arithmetic mistake but it is clear that the student knows the algorithm

$\Rightarrow \gcd(297, 1008) = 9$

2. Way one

$9 = 63 - 54 = 63 - (117 - 63) = 2 \cdot 63 - 117 =$

$= 2(297 - 2 \cdot 117) - 117 = 2 \cdot 297 - 5 \cdot 117 = 2 \cdot 297 - 5 \cdot (1008 - 3 \cdot 297) =$

$= 17 \cdot 297 - 5 \cdot 1008 \Rightarrow \boxed{x=17, y=-5}$

Way two:

$$\left(\begin{array}{cc|c} 1 & 0 & 297 \\ 0 & 1 & 1008 \end{array} \right) \xrightarrow{R_2 \rightarrow R_2 - 3R_1} \left(\begin{array}{cc|c} 1 & 0 & 297 \\ -3 & 1 & 117 \end{array} \right) \xrightarrow{R_1 \rightarrow R_1 - 2R_2} \left(\begin{array}{cc|c} 7 & -2 & 63 \\ -3 & 1 & 117 \end{array} \right)$$

$\xrightarrow{R_2 \rightarrow R_2 - R_1} \left(\begin{array}{cc|c} 7 & -2 & 63 \\ -10 & 3 & 54 \end{array} \right) \xrightarrow{R_1 \rightarrow R_1 - R_2} \left(\begin{array}{cc|c} 17 & -5 & 9 \\ -10 & 3 & 54 \end{array} \right) \Rightarrow 17 \cdot 297 - 5 \cdot 1008 = 9$

$\boxed{x=17, y=-5}$