

NAME (printed neatly)____________________ QUIZ#2 GRADE ________

Directions for taking quizzes: Write your name legibly where indicated on both sides of this paper. On the reverse side of this paper, circle the letter category which corresponds to the first letter of your last name. After you have completed this quiz, fold this paper lengthwise such that the side with your solution is in the inside of the fold (so your quiz grade will be hidden when returning papers.) Turn your quiz in on the appropriate pile as determined by the first latter of your last name. Follow the Aggie Honor Code!

1. Prove by induction that if x is not equal to -1 and n is any positive integer then

$$1 - x + x^2 - \ldots + (-1)^n x^n = \frac{1-(-1)^{n+1}x^{n+1}}{1+x}.$$

Solution

1) Base of induction

For $n=1$ the left handside of the considered formula is $1-x$ and the right handside is

$$\frac{1-x^2}{1+x} = \frac{(1-x)\cancel{(1+x)}}{\cancel{1+x}} = 1-x \Rightarrow$$ the formula holds for $n=1$

2) Assume that the formula holds for $n=k$ and prove it for $n=k+1$

Given: $1-x+x^2+\ldots+(-1)^k x^k = \dfrac{1-(-1)^{k+1}x^{k+1}}{1+x}$

We have to prove: $1-x+x^2+\ldots+(-1)^k x^k + (-1)^{k+1}x^{k+1} = \dfrac{1-(-1)^{k+2}x^{k+2}}{1+x}$

Indeed,

$$1-x+x^2+\ldots+(-1)^{k+1}x^{k+1} = \left(1-x+x^2+\ldots+(-1)^k x^k\right)+(-1)^{k+1}x^{k+1} =$$

$$\underset{\substack{\uparrow \\ \text{By induction} \\ \text{hypothesis}}}{=} \frac{1-(-1)^{k+1}x^{k+1}}{1+x} + (-1)^{k+1}x^{k+1} \underset{\substack{\downarrow \\ \text{Bring to} \\ \text{the common} \\ \text{denominator}}}{=} \frac{1-(-1)^{k+1}x^{k+1}+(-1)^{k+1}x^{k+1}(1+x)}{1+x} =$$

$$= \frac{1-\cancel{(-1)^{k+1}x^{k+1}}+\cancel{(-1)^{k+1}x^{k+1}}+(-1)^{k+1}x^{k+2}}{1+x} = \frac{1-(-1)^{k+2}x^{k+2}}{1+x}$$

q.e.d.