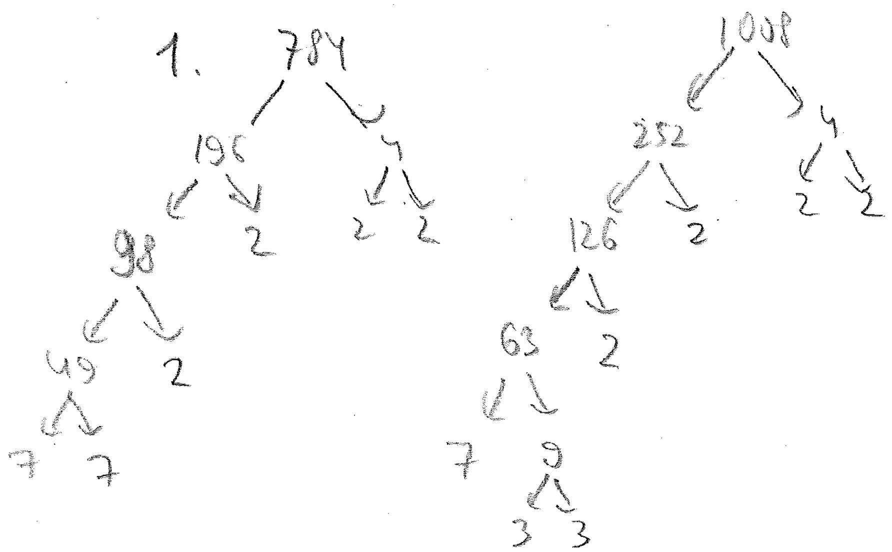
NAME (printed neatly)____________________ QUIZ#3 GRADE ________

Directions for taking quizzes: Write your name legibly where indicated on both sides of this paper. On the reverse side of this paper, circle the letter category which corresponds to the first letter of your last name. After you have completed this quiz, fold this paper lengthwise such that the side with your solution is in the inside of the fold (so your quiz grade will be hidden when returning papers.) Turn your quiz in on the appropriate pile as determined by the first latter of your last name. Follow the Aggie Honor Code!

1. Using the prime factorization, find the greatest common divisor and the least common multiple of 784 and 1008.
2. Show that if $2^n + 1$ is prime, where $n \geq 1$, then n must be of the form 2^k for a positive integer k
3. **(bonus 25 points)** Suppose that a and b are positive integers. Explain why $\gcd(a^2, b^2)$ cannot be equal to 27.

1.



$\Rightarrow$ $784 = 2^4 \cdot 7^2$

$1008 = 2^4 \cdot 3^2 \cdot 7$

$\Downarrow$

$\gcd(784, 1008) = 2^4 \cdot 7 = 16 \cdot 7 = 112$

$\mathrm{lcm}(784, 1008) = 2^4 \cdot 3^2 \cdot 7^2 = 1008 \times 7 = 7056$

2. The statement is equivalent to the following one: if n is not of the form 2^k then 2^n+1 is composite. Prove the last statement. If n is not of the form 2^k then there is odd $p > 1$ s.t. $p \mid n$, i.e. $n = \ell p$ for some integer ℓ

$2^n+1 = 2^{\ell p} + 1 = (2^\ell)^p + 1^p = (2^\ell + 1)\left((2^\ell)^{p-1} - (2^\ell)^{p-2} + (2^\ell)^{p-3} - \dots - 2^\ell + 1\right)$

$1 < 2^\ell + 1 < 2^n + 1 \Rightarrow 2^n + 1$ is composite

-you can continue your solution in the next page, if you do not have enough space

3. The powers of prime in the prime decomposition of a^2 and b^2 are even $\Rightarrow$ the same is true for $\gcd(a^2, b^2)$ but $27 = 3^3$ i.e the power of 3 is odd $\Rightarrow$ 27 cannot be $\gcd(a^2, b^2)$