NAME (printed neatly)_____ QUIZ#4 GRADE _____

Directions for taking quizzes: Write your name legibly where indicated on both sides of this paper. On the reverse side of this paper, circle the letter category which corresponds to the first letter of your last name. After you have completed this quiz, fold this paper lengthwise such that the side with your solution is in the inside of the fold (so your quiz grade will be hidden when returning papers.) Turn your quiz in on the appropriate pile as determined by the first latter of your last name. Follow the Aggie Honor Code!

1. Is $[37]_{51}$ invertible (justify your answer!). If yes, find the inverse (indicate the standard representative of the obtained congruence class).
2. Find the last digit of $3^{1,000,023}$. Explain your answer.

1. [Yes], $[37]_{51}$ is invertible, because 37 and 51 are relatively prime (i.e. $\gcd(37,51)=1$). Indeed the prime factorization are $37=37$, $51=17\times 3$

To find the inverse solve the equation

$$37x+51y=1$$

in integers x and y. Use the matrix form of the Euclidean algorithm:

$$\left(\begin{array}{cc|c} 1 & 0 & 37 \\ 0 & 1 & 51 \end{array}\right) \xrightarrow{R_2\to R_2-R_1} \left(\begin{array}{cc|c} 1 & 0 & 37 \\ -1 & 1 & 14 \end{array}\right) \xrightarrow{R_1\to R_1-2R_2} \left(\begin{array}{cc|c} 3 & -2 & 9 \\ -1 & 1 & 14 \end{array}\right) \xrightarrow{R_2\to R_2-R_1}$$

$$\left(\begin{array}{cc|c} 3 & -2 & 9 \\ -4 & 3 & 5 \end{array}\right) \xrightarrow{R_1\to R_1-R_2} \left(\begin{array}{cc|c} 7 & -5 & 4 \\ -4 & 3 & 5 \end{array}\right) \xrightarrow{R_2\to R_2-R_1} \left(\begin{array}{cc|c} 7 & -5 & 4 \\ -11 & 8 & 1 \end{array}\right) \Rightarrow$$

$$37\times(-11)+51\cdot 8=1 \Rightarrow [37]_{51}=[-11]_{51}=\boxed{[40]_{51}}$$

(40 is the standard representative)

2. The last digit is the remainder of the number, when divided by 10. Note that

$$3^i \equiv \begin{cases} 1 \bmod 10 & \text{if } i\equiv 0 \bmod 4 \\ 3 \bmod 10 & \text{if } i\equiv 1 \bmod 4 \\ 9 \bmod 10 & \text{if } i\equiv 2 \bmod 4 \\ 7 \bmod 10 & \text{if } i\equiv 3 \bmod 4 \end{cases} \Leftarrow \begin{array}{l} 3^0\equiv 1 \bmod 10 \\ 3^1\equiv 3 \bmod 10 \\ 3^2\equiv 9 \bmod 10 \\ 3^3\equiv 7 \bmod 10 \\ 3^4\equiv 1 \bmod 10 \end{array}$$

$1,000,023 \equiv 3 \bmod 4 \Rightarrow$ the last digit is $\boxed{7}$