

NAME (printed neatly) _____ QUIZ#5 GRADE _____

Directions for taking quizzes: Write your name legibly where indicated on both sides of this paper. On the reverse side of this paper, circle the letter category which corresponds to the first letter of your last name. After you have completed this quiz, fold this paper lengthwise such that the side with your solution is in the inside of the fold (so your quiz grade will be hidden when returning papers.) Turn your quiz in on the appropriate pile as determined by the first letter of your last name. Follow the Aggie Honor Code!

- (a) Find all the solutions (if there are any) of the linear congruence $35x \equiv 21 \pmod{56}$.
 (b) Based on your solution of item (a) write all congruence classes modulo 56 that satisfy the linear congruence of item (a).
- Solve the following set of simultaneous linear congruences: $x \equiv 3 \pmod{8}$ and $x \equiv 5 \pmod{13}$.

1. (a) $\gcd(35, 56) = 7$ and it divides 21 \Rightarrow the congruence is solvable
 $35x \equiv 21 \pmod{56} \Rightarrow 5x \equiv 3 \pmod{8} \Rightarrow [x]_8 = [5]_8^{-1} [3]_8$
 Find $[5]_8^{-1} \begin{pmatrix} 1 & 0 & | & 5 \\ 0 & 1 & | & 8 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & | & 5 \\ -1 & 1 & | & 3 \end{pmatrix} \sim \begin{pmatrix} 2 & -1 & | & 2 \\ -1 & 1 & | & 3 \end{pmatrix} \sim \begin{pmatrix} 2 & -1 & | & 2 \\ -3 & 2 & | & 3 \end{pmatrix} \Rightarrow$
 $5 \cdot (-3) + 8 \cdot 2 \equiv 1 \Rightarrow [5]_8^{-1} = [-3]_8 = [5]_8 \Rightarrow$
 $x = [5]_8 [3]_8 = [15]_8 = [7]_8 \Rightarrow \boxed{x \equiv 7 \pmod{8}}$
 (b) We have 7 congruence classes mod 56: $[7]_{56}, [15]_{56}, [23]_{56}, [31]_{56},$
 $[39]_{56}, [47]_{56}, [55]_{56}$

2. 8 and 13 are coprime \Rightarrow we can use the Chinese Remainder Thm
 Find k and t such that $8k + 13t = 1$
 $\begin{pmatrix} 1 & 0 & | & 8 \\ 0 & 1 & | & 13 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & | & 8 \\ -1 & 1 & | & 5 \end{pmatrix} \sim \begin{pmatrix} 2 & -1 & | & 3 \\ -1 & 1 & | & 5 \end{pmatrix} \sim \begin{pmatrix} 2 & -1 & | & 3 \\ -3 & 2 & | & 2 \end{pmatrix} \sim \begin{pmatrix} 5 & -3 & | & 1 \\ -3 & 2 & | & 2 \end{pmatrix} \Rightarrow$
 $k = 5, t = -3 \Rightarrow c = 8k + 13t = 8 \cdot 5 + 13 \cdot (-3) = 40 - 39 = 1$
 $= 200 - 117 = 83$ is a solution mod $13 \times 8 = 104 \Rightarrow$
 $\boxed{x \equiv 83 \pmod{104}}$

-you can continue your solution in the next page, if you do not have enough space