NAME (printed neatly)____________________ QUIZ#6 GRADE ________

Directions for taking quizzes: Write your name legibly where indicated on both sides of this paper. On the reverse side of this paper, circle the letter category which corresponds to the first letter of your last name. After you have completed this quiz, fold this paper lengthwise such that the side with your solution is in the inside of the fold (so your quiz grade will be hidden when returning papers.) Turn your quiz in on the appropriate pile as determined by the first latter of your last name. Follow the Aggie Honor Code!

1. (a) Find $\varphi(1000)$;
   (b) Find the last 3 digits of 9^{803}.
2. Show that, for every integer n, $n^{13} - n$ is divisible by 2, 3, 5, 7, and 13.

1 (a) $1000 = 2^3 \cdot 5^3 \Rightarrow \varphi(1000) = (2^3-2^2)(5^3-5^2) = (8-4)(125-25) = \boxed{400}$

(b) $803 \equiv 3 \mod 400 \Rightarrow$

$9^{803} \equiv 9^3 = 729 \mod 1000 \Rightarrow$ the last 3 digits are $\boxed{729}$

2. $n^{13} - n = n(n^{12}-1)$ (*)

Let p is one of the prime numbers 2, 3, 5, 7, or 13.

i) If n is divisible by p then from (*) $n^{13}-n$ is divisible by p

ii) If n is not divisible by p then by the Fermat theorem

$$n^{p-1} \equiv 1 \mod p \quad (**)$$

Not that for any $p \in \{2,3,5,7,13\}$ $p-1$ is the divisor of 12 $\Rightarrow$

(**) implies that $n^{12} \equiv 1 \mod p$ i.e. $p \mid (n^{12}-1) \Rightarrow p \mid n^{13}-n$

q.e.d.

-you can continue your solution in the next page, if you do not have enough space