

NAME (printed neatly) _____ QUIZ#9 GRADE _____

Directions for taking quizzes: the same as in the previous quizzes.

1. Let $\pi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 3 & 5 & 1 & 4 & 9 & 7 & 2 & 6 \end{pmatrix}$ and $\pi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 6 & 4 & 3 & 2 & 9 & 8 & 1 & 5 \end{pmatrix}$

Find $\pi_1\pi_2$ and $\pi_2\pi_1$.

2. Let π is the following product of cycles: $\pi = (1459)(12378)(356)(2579)$

(a) Represent π as a product of (pairwise) disjoint cycles;

(b) Find the order of π .

3. Give an example of a permutation of order 2 which is not a transposition.

1. $\pi_1 \pi_2$:

$$\begin{matrix} \pi_2 \\ \pi_1 \end{matrix} \left\{ \begin{matrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 6 & 4 & 3 & 2 & 9 & 8 & 1 & 5 \end{pmatrix} \\ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 3 & 5 & 1 & 4 & 9 & 7 & 2 & 6 \end{pmatrix} \end{matrix} \right\} \Rightarrow \boxed{\pi_1\pi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 9 & 1 & 5 & 3 & 6 & 2 & 8 & 4 \end{pmatrix}}$$

$$\begin{matrix} \pi_2\pi_1 \\ \pi_1 \\ \pi_2 \end{matrix} \left\{ \begin{matrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 3 & 5 & 1 & 4 & 9 & 7 & 2 & 6 \end{pmatrix} \\ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 6 & 4 & 3 & 2 & 9 & 8 & 1 & 5 \end{pmatrix} \end{matrix} \right\} \Rightarrow \boxed{\pi_2\pi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 4 & 2 & 7 & 3 & 5 & 8 & 6 & 9 \end{pmatrix}}$$

2. a) $\pi = \underbrace{(1459)}_{\tau_1} \underbrace{(12378)}_{\tau_3} \underbrace{(356)}_{\tau_2} \underbrace{(2579)}_{\tau_4}$

i) $\left. \begin{matrix} 1 \xrightarrow{\tau_3\tau_2\tau_1} 2 \xrightarrow{\tau_4} 2 \Rightarrow 1 \xrightarrow{\pi} 2 \\ 2 \xrightarrow{\tau_1} 5 \xrightarrow{\tau_2} 6 \xrightarrow{\tau_4\tau_3} 6 \Rightarrow 2 \xrightarrow{\pi} 6 \\ 6 \xrightarrow{\tau_2\tau_1} 3 \xrightarrow{\tau_3} 7 \Rightarrow 6 \xrightarrow{\pi} 7 \\ 7 \xrightarrow{\tau_1} 9 \xrightarrow{\tau_4\tau_3\tau_2} 1 \Rightarrow 7 \xrightarrow{\pi} 9 \end{matrix} \right\} \text{cycle } (1267)$

ii) $\left. \begin{matrix} 3 \xrightarrow{\tau_2\tau_1} 5 \xrightarrow{\tau_4\tau_3} 9 \Rightarrow 3 \xrightarrow{\pi} 9 \\ 9 \xrightarrow{\tau_1} 2 \xrightarrow{\tau_3\tau_2} 3 \xrightarrow{\tau_4} 3 \Rightarrow 9 \xrightarrow{\pi} 3 \end{matrix} \right\} \text{cycle } (3,9)$

iii) $\left. \begin{matrix} 4 \xrightarrow{\pi} 5 \\ 5 \xrightarrow{\tau_1} 7 \xrightarrow{\tau_3} 8 \xrightarrow{\tau_2} 8 \Rightarrow 5 \xrightarrow{\pi} 8 \\ 8 \xrightarrow{\tau_3\tau_2\tau_1} 1 \xrightarrow{\tau_4} 4 \Rightarrow 8 \xrightarrow{\pi} 4 \end{matrix} \right\} \text{cycle } (458) \Rightarrow \pi = (1267)(39)(458)$

-you can continue your solution in the next page, if you do not have enough space

See the back page

NAME _____

Circle First Letter of Last Name

A-F G-K L-O P-Z

$$b) o(\pi) = \text{lcm}(4, 2, 3) = \boxed{12}$$

3. A product of any two disjoint transpositions is not a transposition and has order 2

For example $n \geq 4$,

$$\pi = (12)(34)$$