

Topics for exam 2, MATH308-FALL 2015

1. From chapter 7: sections 7.1-7.6 and section 7.8 The main points to know are:
 - (a) how to transform a scalar differential equation of higher order to a system of differential equations of the first order;
 - (b) how to transform a system of differential equations to a matrix form;
 - (c) what is a fundamental set of solutions of a first order linear homogeneous system of differential equations and how to check that the given set of solutions is fundamental (section 7.4);
 - (d) what are eigenvalues and eigenvectors of a given matrix and how to find them (section 7.3);
 - (e) In the case of linear systems of differential equations with constant coefficients and distinct real eigenvalues (most of the section 7.5) pay your attention on *how on the base of the knowledge of the eigenvalues and eigenvectors to form a fundamental set of solutions and to write the general solution.*
 - (f) In the case of complex eigenvalues (section 7.6, mainly the cases $n = 2$ and $n = 3$) make sure that you understand how from a complex valued solution to obtain the real-valued solutions;
 - (g) In the case of repeated eigenvalues (section 7.8) you have also to know the definition of matrix exponential from section 7.7 and to understand how to use the formula

$$e^{tA}x = e^{\lambda t} e^{t(A-\lambda I)}x. \quad (1)$$

In particular, for this case you have

- to know how to find the algebraic and geometric multiplicities of an eigenvalue (see also page 385-386 of the 10th edition or page 380 of the 9th edition of the textbook);
 - based on the knowledge of algebraic and geometric multiplicities of the eigenvalues to know how to determine whether a basis of eigenvectors exists;
 - in the case when it does not exist to know how to find the basis of generalized eigenvectors in different situations;
 - to know how, after finding the basis of generalized eigenvectors, to write down the general solution of the original system in terms of these generalized eigenvectors based of formula (1) (note that the case of eigenvectors of algebraic multiplicity 3 will not be given in the test);
- (h) on the base of knowledge of the general solution of a system of differential equation, to find the solution of a given initial value problem.¹
2. Section 9.1: The phase portrait of planar linear systems: to know all types of critical (equilibrium) points and their stability property (for the summary see Table 9.1.1 on page 504 of the 10th edition or on page 494 of the 9th edition of the book); also to know to sketch the corresponding phase portraits.

It is recommended to review all problems in homework assignments 8-12 and the examples given during the class on the topics listed above. In addition, it is useful to review homework assignments 12-15 and 17 of Spring 2015 term posted at <http://www.math.tamu.edu/zelenko/S15308Hmwk.html> (except problem 2 of homework assignment 17 there; also in homework 15 there in the formulations of the problems 1 and 3 fundamental matrices can be replaced by fundamental set of solutions) and homework assignments 14-17 of Fall 2013 term posted at <http://www.math.tamu.edu/zelenko/F13308Hmwk.html> Note that in the previous semesters I explained the topic of section 7.8 in a different way, so some parts of the explanations in the solutions of the previous semesters (namely when we pass from the knowledge of the basis of generalized eigenvectors to the general solutions) can be omitted.

For repeated eigenvalues, you have to make sure that in the case of an eigenvalue of algebraic multiplicity 2 you are able to distinguish the subcase of geometric multiplicity 2 and the subcase of geometric multiplicity 1 and to know how to find the fundamental set of solution in each of these subcases. For this I recommend you to review the class notes and the corresponding material posted on "Lecture notes and enrichment" in our course webpage. Note that in this material you can ignore the first page (this material was posted last semester when I explained this topic in a different way; this semester I explained this material in class exactly as in these files starting from page 2; also at the end of page 8 in these files I consider the case when λ_1 has the algebraic multiplicity 2 and the geometric multiplicity 1 (this is not stated explicitly there).

¹especially in the case $n = 3$ you have to practice the Gauss elimination method.