Topics for exam 2, MATH308-FALL 2015

1. From Chapter 3:
   (a) Linear homogeneous equations of second order with constant coefficient in all possible cases (two distinct real roots, complex roots, repeated root) (sections 3.1, 3.3, and 3.4);
   (b) Spring-mass systems: undamped free vibration (including finding of the amplitude, the natural frequency, the period and the phase).
   (c) Spring-mass systems: damped free vibration, including finding of quasi-period, and understanding the meaning of critical damping constant.

   The last two items correspond to section 3.7. You also have to be able in both cases to sketch the graph of the solutions based on the method of parent functions.

2. From chapter 7: sections 7.1-7.6, the notion of matrix exponential of section 7.7, and section 7.8. The main points to know are:
   (a) how to transform a scalar differential equation of higher order to a system of differential equations of the first order;
   (b) how to transform a system of differential equations to a matrix form.
   (c) what is a fundamental set of solutions of a first order linear homogeneous system of differential equations and how to check that the given set of solutions is fundamental (section 7.4).
   (d) what are eigenvalues and eigenvectors of a given matrix and how to find them (section 7.3).
   (e) how to solve a system of differential equations and IVP[1] in the case of distinct real eigenvalues (section 7.5).

   Especially pay your attention here on *how on the base of the knowledge of the eigenvalues and eigenvectors to form a fundamental set of solutions and to write the general solution.*

   (a) Linear systems of differential equations with constant coefficients and complex eigenvalues (in the cases $n = 2$ and $n = 3$) (section 7.6);
   (b) Section 7.8: Linear systems of differential equations with constant coefficients and repeated eigenvalues (in the cases of $n = 2$ and $n = 3$). For this you have also to know the definition of matrix exponential from section 7.7 and to understand and to know how to use the formula

   $$e^{tA}x = e^{\lambda t}e^{t(A-\lambda I)}x. \tag{1}$$

   In particular, you have
      i. to know how to find the algebraic and geometric multiplicities of an eigenvalue discussed in class (see also page 385-386 of 10th edition or page 385 of 9th edition of the textbook);
      ii. based on the knowledge of algebraic and geometric multiplicities of the eigenvalues to know how to determine whether a basis of eigenvectors exists;
      iii. in the case when it does not exist to know how to find the basis of generalized eigenvectors in different situations;
      iv. to know how, after finding the basis of generalized eigenvectors, to write down the general solution of the original system in terms of these generalized eigenvectors based of formula (1) (note that the case of eigenvectors of algebraic multiplicity 3 will not be given in the test).

It is *recommended to review all problems in homework assignments 5–9 and the examples given during the class on the topics listed above. In addition, it is useful to review homework assignments 6–11 of Fall 2015 term posted at at http://www.math.tamu.edu/ zelenko/F15308Hmwk.html*

Especially, for repeated eigenvalues, you have to make sure that in the case of an eigenvalue of algebraic multiplicity 2 you are able to distinguish the subcase of geometric multiplicity 2 and the subcase of geometric multiplicity 1 and to know how to find the fundamental set of solution in each of these subcases. For this I recommend you to review your lecture notes and the corresponding material posted on "Lecture notes and enrichment" in our course webpage. Note that in this material you can ignore the first page (this material was posted in Spring 2015 semester when I explained this topic in a different way; this semester I explained this material in class exactly as in these files starting from page 2; also at the end of page 8 in these files I consider the case when λ_1 has algebraic multiplicity 2 and algebraic multiplicity 1).

[1] especially in the case $n = 3$ you have to practice the Gauss elimination method.