

## Topics for exam 2, MATH308-Spring 2017

The test will be on the material of my lectures notes, sections 15-25. Below the corresponding topics from the book:

1. From Chapter 3:

- (a) Spring-mass systems: undamped free vibration (including finding of the amplitude, the natural frequency, the period and the phase).
- (b) Spring-mass systems: damped free vibration, including finding of quasi-period, and understanding the meaning of critical damping constant.

The last two items correspond to section 3.7. You also have to be able in both cases to sketch the graph of the solutions based on the method of parent functions.

2. Euler's equation (my lecture notes, section 16, Book: beginning of sections 5.4 and problem 34 page 166 of the book)

3. From chapter 7: sections 7.1-7.6, the notion of matrix exponential of section 7.7, and section 7.8 (the case  $n=2$  only). The main points to know are:

- (a) how to transform a scalar differential equation of higher order to a system of differential equations of the first order;
- (b) how to transform a system of differential equations to a matrix form.
- (c) what is a fundamental set of solutions of a first order linear homogeneous system of differential equations and how to check that the given set of solutions is fundamental (section 7.4).
- (d) what are eigenvalues and eigenvectors of a given matrix and how to find them (section 7.3).
- (e) how to solve a system of differential equations and IVP in the case of distinct real eigenvalues (section 7.5) for the cases  $n = 2$  and  $n = 3$ .<sup>1</sup>
- (f) how to solve linear systems of differential equations with constant coefficients and complex eigenvalues in the cases  $n = 2$  and  $n = 3$  (section 7.6);
- (g) Definition of matrix exponential and how to calculate it for some simple classes of matrices such as diagonal matrices or matrices like in problem 3 of homework assignment 7.
- (h) how to solve linear systems of differential equations with constant coefficients and repeated eigenvalues (**in the cases of  $n = 2$  only**).
- (i) For the case of repeated eigenvalues, although the case  $n = 2$  is quite simple without any big theory, it is still beneficial to understand the notion of algebraic and geometric multiplicities of the eigenvalues, of the generalized eigenvector, and how to use the identity

$$e^{tA}x = e^{\lambda t} e^{t(A-\lambda I)}x. \quad (1)$$

In all case of systems above pay your attention on *how on the base of the knowledge of the eigenvalues and eigenvectors/ generalized eigenvectors to form a fundamental set of solutions and to write the general solution.*

It is recommended to review all problems in homework assignments 5–7 and the lecture notes 15-25, including all examples. In addition, it is useful to review homework assignments 5 (problem 5), 6-8, and 9 (problem 1 there) of Fall 2016 term posted at <http://www.math.tamu.edu/zelenko/F16308Hmwk.html>

---

<sup>1</sup>in the case  $n = 3$  for IVP you have to practice the Gauss elimination method.