

Topics for final exam, MATH308-FALL 2016

- (1) Section 9.1: The phase portrait of planar linear systems: to know all types of critical (equilibrium) points and their stability property (see Table 9.1.1 on page 504 and Table 9.1.9, page 507 for the summary); Also to know to sketch the corresponding phase portraits. In particular, to make an accurate sketch you have:
 - (a) to determine and sketch the eigenlines in the case of a node and a saddle;
 - (b) to determine the direction of rotation (clockwise or counterclockwise) in the case of the spiral points (i. e. if $v = a + ib$ is an eigenvector corresponding to the eigenvalues $\lambda = \alpha + i\beta$ with $\beta > 0$, then the rotation is from b to a in the shortest way) ;
 - (c) to determine how the trajectories enter/leave the improper node according to the following rule: if v is an eigenvector and w is a generalized eigenvector such that $(A - \lambda I)w = v$, then
 - (i) if $\lambda > 0$ then the trajectories leave the improper node to the second and fourth quadrants with respect to (v, w) ;
 - (ii) if $\lambda < 0$ then the trajectories enter the improper node from the first and third quadrants with respect to (v, w) .
- (2) Section 9.3: To know how to find critical points of a given nonlinear (but locally linear) system, how to find the linear system corresponding to each critical point, and how to determine the stability properties of the critical point on the base of the corresponding linear system (as in Table 9.1.9, page 507).
- (3) Method of variation of parameters for linear non-homogeneous system of equation of first order (section 7.9) and for linear non-homogeneous equations of 2nd order (sect. 3.6).
- (4) Method of undetermined coefficients for for linear non-homogeneous equations of 2nd order (section 3.5, especially understand carefully Table 3.5.1 on page 182).
- (5) Forced vibration (including finding of *steady state solution*) (section 3.8).
- (6) Laplace Transform (Chapter 6, sections 6.1-6.4). The table of Laplace transform as in page 321 of the textbook will be given. It is recommended to practice to use this table. Even more important is that you have to be aware that before using the table you will have to make an appropriate work based on the principles and techniques you learned. Please make sure to review them. Do not rely on the presence of the table. The table will be useless if you do not know appropriate principles and techniques.

The main points to know for this goal are

- how to transform a given linear differential equation with constant coefficients to an algebraic one using the Laplace transform (section 6.2).
- how to find the inverse Laplace transform of a rational function using the partial fraction decomposition (section 6.2).
- how to find the Laplace transform for a piecewise continuous function (with the help of steps functions, sections 6.3)
- how to apply the items above in order to solve initial value problems for linear non-homogeneous equations (sections 6.2 and 6.4)

It is recommended to review all problems in homework assignments 10-13 and the examples given during the class on the topics listed above. In addition, review homework assignments 12-16 of Fall 2015, except problems 3 and 5 in the assignment 16, posted at <http://www.math.tamu.edu/zelenko/F15308Hmwk.html>