Homework #2 MATH 308 Solutions of additional problems for Honors section.

Problem 2(c) The difference here is that the volume of the water in the pond changes in time and satisfies

$$V'(t) = 10 - 9 = 1 \Rightarrow V(t) = V(0) + t = 200 + t$$

Therefore rate out $= \frac{Q(t)}{V(t)} r_{out} = \frac{9}{200+t} Q(t)$

(rate in is the same as in the previous setting)

$\Rightarrow$ IVP is written as

$$\begin{cases} Q'(t) = 2 + 4\sin 4t - \frac{9}{200+t} Q(t) \\ Q(0) = 60 \end{cases}$$

Problem 3 $ty' - 4y = 2t^2\sqrt{y} \Leftrightarrow t y^{-1/2} y' - 4y^{1/2} = 2t^2$ (1)

In this case $\alpha = 1/2$ and the substitution is $\boxed{v = y^{1/2}} \Rightarrow$

$v' = \frac{1}{2} y^{-1/2} y'$. So substituting to (1) we get

$$2tv' - 4v = 2t^2 \Leftrightarrow v' - \frac{2}{t} v = t$$

This is a linear equation that can be solved by the method of integrating factor: $p = -\frac{2}{t} \Rightarrow \mu' = -\frac{2}{t}\mu \Rightarrow \mu = e^{-\int \frac{2}{t} dt} = e^{-2\ln t} = t^{-2}$

$$(t^{-2} v)' = \frac{1}{t} \Rightarrow t^{-2} v = \ln t + C \Rightarrow v = t^2 \ln t + Ct^2 =$$

$$= t^2 (\ln t + C)$$

② $\boxed{y(t) = (v(t))^2 = t^4(\ln t + C)^2}$

Problem 6 (b) $e^t \sec y - t\tan y + \frac{dy}{dt} = 0$

Multiply by $e^{at}\cos y$: $e^{(a+1)t} - e^{at}\sin y + e^{at}\cos y \frac{dy}{dt} = 0$

$P = e^{(a+1)t} - e^{at}\sin y \qquad P_y = -e^{at}\cos y$ $\Rightarrow$

$Q = e^{at}\cos y \qquad Q_t = a e^{at}\cos y$

$P_y = Q_t \Leftrightarrow \boxed{a = -1}$

In this case: $P = -e^{-t}\sin y + 1$

$Q = e^{-t}\cos y$

$$\begin{cases} \varphi_t = -e^{-t}\sin y + 1 \\ \varphi_y = e^{-t}\cos y \end{cases} \Rightarrow \varphi = \int e^{-t}\cos y\, dy + h(t) =$$

$$= e^{-t}\sin y + h(t) \Rightarrow$$

$\varphi_t = \frac{d}{dt}(e^{-t}\sin y + h(t)) = -e^{-t}\sin y + h'(t) = -e^{-t}\sin y + 1 \Rightarrow$

$h'(t) = 1 \Rightarrow$ we can take $h(t) = t \Rightarrow$

$\varphi = e^{-t}\sin y + t \Rightarrow$ general solution is

$e^{-t}\sin y + t = C$ or $\sin y = (C - t)e^t \Rightarrow$

$y = \arcsin((C-t)e^t)$