Homework Assignment 10 in Differential Equations, MATH308-Spring 2017, Honors section

Bonus homework (strongly recommended to do anyway, the material is a part of the final exam),

due May 3 by 3 p.m., place it into my mailbox on the second floor of Blocker

Topics covered : *Stability and locally linear systems (section 9.2 and 9.3); the method of variation of parameters for nonhomogeneous systems of first order and differential equations of second order (section 7.9 and 3.6).*

1. Given the following system of differential equations $\begin{cases} x' &= \dfrac{xy-1}{2} \\ y' &= 4x - \dfrac{1}{4}y^3. \end{cases}$

   (a) Determine all critical points.

   (b) Find the corresponding linear system near each critical point.

   (c) Determine the type of the critical point of each linear systems obtained in the previous item. On the base of this information what conclusions (if any) can be given on the stability properties of the corresponding critical points of the original system , found in item a) (whether they are stable, asymptotically stable, or unstable)?

2. For the following system

$$\begin{cases} x' &= x(22-5x-7y) \\ y' &= y(9-3y-2x) \end{cases} \tag{1}$$

   (a) Determine all critical points.

   (b) Find the corresponding linear system near each critical point.

   (c) Based on this linear systems determine the type of each critical point and their stability properties (i.e. whether they are stable, asymptotically stable, or unstable)?

   (d) Sketch the phase portrait of system (1) in the first quadrant.

   (e) System (1) corresponds to a model of competing species. Review the end of class notes of April 25 (pages 18-190, regarding an example of a model of competing species and section 9.4 in the textbook. Based on your analysis in the previous item, answer the following question: does the coexistence occurs in the model given by system (1)?

3. Use the method of variation of parameters to find the genral solution of the following nonhomogeneous linear systems/differential equations:

   (a) $\begin{cases} x_1' = 5x_1 + 4x_2 + (t+2)e^{-2t} \\ x_2' = -2x_1 - 4x_2 - 3e^{-2t} \end{cases}$;

   (b) $y'' - 4y' + 4y = \dfrac{e^{2t}}{1+t^2}$;

   (c)

$$\mathbf{x}' = \begin{pmatrix} 15 & 8 & -28 \\ 7 & 1 & -10 \\ 9 & 4 & -17 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 2e^{-3t} \\ e^{-3t} \\ -2e^{-3t} \end{pmatrix}, \quad x \in \mathbb{R}^3.$$