

Homework Assignment 11 in Differential Equations, MATH308-FALL 2015

due November 21, 2016

Sections covered: 7.9 & 3.6 (the method of variation of parameters for nonhomogeneous systems of first order and differential equations of second order) and 3.5 (regarding the method of undetermined coefficients for nonhomogeneous differential equations of second order with right hand-side of the form $e^{\alpha t}P_n(t)$, corresponding to the first two lines of the table 3.5.1 of page 182 of the textbook)

1. Use the method of variation of parameters to find the solution of the following initial value problems:

(a)

$$\begin{cases} x_1' = 2x_1 - 5x_2 + \sin t \\ x_2' = x_1 - 2x_2 + \tan t \end{cases}, \quad x_1(0) = 0, \quad x_2(0) = 0.$$

(b)

$$2y'' - 3y' + y = e^t(1 + t^2), \quad y(0) = 3, \quad y'(0) = -4.$$

(c)

$$\mathbf{x}' = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} e^t, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

2. (a) For each of the following equations write down the form in which a particular solution should be found according to the method of undetermined coefficients (**you do not need to find the value of the undetermined coefficient/coefficients in this item**):

i. $y'' + 3y' - 18y = t^4 + (t^2 + 3)e^{-t} + te^{-6t} + (t^4 + t^2)e^{3t}$;

ii. $y'' + 10y' + 25y = e^{-5t} + (t + 1)e^{3t}$

- (b) Find the general solution of the equation from item (a)ii above (using the method of undetermined coefficients).