Homework Assignment 12 in Differential Equations, MATH308-Fall 2016

due November 30, 2016

Topics covered : *The method of undetermined coefficients for second order equations: the case of the right-hand side of the form $P_n(t)e^{\alpha t}\sin(\beta t)$ or $P_n(t)e^{\alpha t}\cos(\beta t)$ (chapter 3.5, table 3.5.1. on page 182, especially the third line there); applications to forced vibration (section 3.8), use that the gravitational acceleration $g = 32\frac{ft}{s^2}$; Laplace transform: inverse Laplace transform of rational functions using partial fraction decomposition; solution of initial value problems using Laplace transform(corresponds to sections 6.1 and beginning of section 6.2)*

1. For each of the following equations write down the form in which a particular solution should be found according to the method of undetermined coefficients (**you do not need to find the value of the undetermined coefficient/coefficients here**):
   (a) $y'' - 6y' + 13y = 3t^4e^{3t} - (t^3+1)e^{2t}\sin(3t) + (t^2-1)e^{2t} - t^2e^{3t}\cos(2t)$;
   (b) $y'' - 10y' + 21y = t^2e^{3t}\sin(7t) - 5te^{7t}\cos(3t) + 5t^3e^{3t} - 4te^{3t} + t^2e^{5t}$;
   (c) $y'' + 16y' + 64y = te^{-8t}\cos(10t) + te^{-8t} - t^3e^{8t}$.
2. A spring is stretched 2 in by a mass that weighs 4 lb. The mass is attached to a dashpot mechanism that has a damping constant of $4\frac{lb\cdot s}{ft}$ and is acted on by an external force of $2\cos 4t + 4\sin 4t$ lb. Determine the steady state solution of this system, using the method of undetermined coefficients.
3. Find the inverse Laplace transform of the following functions:
   (a) $\dfrac{s^2-1}{(s^2-4s+4)(s+4)}$;
   (b) $\dfrac{4s+5}{(s-5)(9s^2+18s+90)}$;
   (c) (**bonus 20 points**) $\dfrac{s^3}{(s^2+9)^2}$. (the material in http://www.math.tamu.edu/ zelenko/ODELaplacecomplex.pd on partial fraction decomposition using complex root might be useful to solve this problem)
4. Using the method of Laplace transform solve the following initial value problem:

$$y'' - 2y' + 10y = 5\sin t, \quad y(0) = -1,\ y'(0) = 1.$$