

Homework Assignment 12 in Differential Equations, MATH308-SPRING 2015

due April 1, 2015

Sections covered 7.1, 7.2, 7.4

1. Let $A = \begin{pmatrix} 5 & -3 \\ -2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -6 \\ -2 & 3 \end{pmatrix}$. Compute $AB - BA$.

2. Transform the given equation into a system of first order differential equations:

(a) $u'' - 5u' + 8u = \tan t$

(b) $y^{(3)} + 4y' - 5ty = 0$

3. Express the given system of linear differential equations in matrix form:

(a)
$$\begin{cases} x_1' &= 3x_1 + 4x_3 \\ x_2' &= -x_1 + 2x_2 - 3x_3 \\ x_3' &= x_1 - x_3 \end{cases}$$

(b)
$$\begin{cases} x' &= \cos t x + t^2 y - \frac{t^2}{2} \\ y' &= -t^2 x - \sin t y + \frac{t^3}{3} \end{cases}$$

4. Determine whether the following solutions of the the system $x'(t) = Ax(t)$ form a fundamental set of its solutions. If they do, give a general solution of the system.

(a) $x^1 = e^{-3t} \begin{pmatrix} -2 \\ 10 \end{pmatrix}, \quad x^2 = e^{-3t} \begin{pmatrix} 3 \\ -15 \end{pmatrix}$

(b) $x^1 = \begin{pmatrix} e^{-2t} \\ -2e^{-2t} \\ 3e^{-2t} \end{pmatrix}, \quad x^2 = \begin{pmatrix} -2 \cos 3t \\ -3 \sin 3t \\ \sin 3t \end{pmatrix}, \quad x^3 = \begin{pmatrix} -2 \sin 3t \\ 3 \cos 3t \\ -\cos 3t \end{pmatrix}$