Homework Assignment # 13 Solutions MATH 308-505

Problem 1

$$AB - BA = \begin{pmatrix} 5 & -3 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 4 & -6 \\ -2 & 3 \end{pmatrix} - \begin{pmatrix} 4 & -6 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 5 & -3 \\ -2 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 20+6 & -30-9 \\ -8-2 & 12+3 \end{pmatrix} - \begin{pmatrix} 20+12 & -12-6 \\ -10-6 & 6+3 \end{pmatrix} =$$

$$= \begin{pmatrix} 26 & -39 \\ -10 & 15 \end{pmatrix} - \begin{pmatrix} 32 & -18 \\ -16 & 9 \end{pmatrix} = \begin{pmatrix} -6 & -21 \\ 6 & 6 \end{pmatrix}$$

Problem 2

a) Let $u_1 = u$, $u_2 = u'$ $\Rightarrow$

$u_1' = u_2$

$u_2' = u'' = 5u' - 8u + \tan t = 5u_2 - 8u_1 + \tan t \Rightarrow$ the required system is

$$\begin{cases} u_1' = u_2 \\ u_2' = -8u_1 + 5u_2 + \tan t \end{cases}$$

b) $y^{(3)} + 4y' - 5ty = 0$

Let $y_1 = y$, $y_2 = y'$, $y_3 = y'' \Rightarrow$

$y_1' = y_2$

$y_2' = y_3$

$y_3' = y^{(3)} = -4y' + 5ty = 5ty_1 - 4y_2 \Rightarrow$ the required system

$$\begin{cases} y_1' = y_2 \\ y_2' = y_3 \\ y_3' = 5ty_1 - 4y_2 \end{cases}$$

Problem 3

(a) $$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 3 & 0 & 4 \\ -1 & 2 & -3 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

(b) $$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos t & t^2 \\ -t^2 & -\sin t \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -\frac{t^2}{2} \\ \frac{t^3}{3} \end{pmatrix}$$

Problem 4 a) Check the Wronskian of x^1 and x^2

$$\begin{vmatrix} -2e^{-3t} & 3e^{-3t} \\ 10e^{-3t} & -15e^{-3t} \end{vmatrix} = (30-30)e^{-6t} = 0 \Rightarrow x^1, x^2 \text{ is } \underline{\text{not}}$$

a fundamental set of solutions

(b) Check the wronskian of x^1, x^2, x^3

$$\begin{matrix} + \\ - \\ + \end{matrix}\begin{vmatrix} e^{-2t} & -2\cos 3t & -2\sin 3t \\ -2e^{-2t} & -3\sin 3t & 3\cos 3t \\ 3e^{-2t} & \sin 3t & -\cos 3t \end{vmatrix} =$$

expand w.r.t. the <u>first column</u>:

$$= e^{-2t} \begin{vmatrix} -3\sin 3t & 3\cos 3t \\ \sin 3t & -\cos 3t \end{vmatrix} - (-2e^{-2t}) \begin{vmatrix} -2\cos 3t & -2\sin 3t \\ \sin 3t & -\cos 3t \end{vmatrix} +$$

$$+ 3e^{-2t} \begin{vmatrix} -2\cos 3t & -2\sin 3t \\ -3\sin 3t & 3\cos 3t \end{vmatrix} = e^{-2t}(3\sin 3t\cos 3t - 3\cos 3t\cos 3t) +$$

$$+ 2e^{-2t}(2\cos^2 3t + 2\sin^2 3t) + 3e^{-2t}(-6\cos^2 3t - 6\sin^2 3t) =$$

$$= 4e^{-2t} - 18e^{-2t} = -14e^{-2t} \neq 0 \Rightarrow$$

x^1, x^2 and x^3 is a fundamental set of solutions

Rem: You could check if the Wronskian is not zero at one (convenient) time moment (for example, when $t=0$) then it is automatically zero for any time moment.

$\Rightarrow$ Gen. solution is

$$c_1 \begin{pmatrix} e^{-2t} \\ -2e^{-2t} \\ 3e^{-2t} \end{pmatrix} + c_2 \begin{pmatrix} -2\cos 3t \\ -3\sin 3t \\ \sin 3t \end{pmatrix} + c_3 \begin{pmatrix} -2\sin 3t \\ 3\cos 3t \\ -\cos 3t \end{pmatrix}$$