ge 1) Homework # 13 solution MATH 308

Problem 1 Find the Laplace transform of the function

$$f(t) = \begin{cases} 2t-1 & t<3 \\ t^3-2t+3 & 3\le t<4 \\ 1-2t & t\ge 4 \end{cases}$$

Solution

$$f(t) = (2t-1) + \left(t^3-2t+3-(2t-1)\right) u_3(t) + \left(1-2t-(t^3-2t+3)\right) u_4(t) = 2t-1 + (t^3-4t+4)\, u_3(t) + (-t^3-2)\, u_4(t)$$

- $\mathcal{L}\{2t-1\} = 2\frac{1}{s^2} - \frac{1}{s}$

- $\mathcal{L}\{(t^3-4t+4)\, \underset{c=3}{u_3(t)}\} = e^{-3s}\, \mathcal{L}\{(t+3)^3 - 4(t+3)+4\} =$

$$= e^{-3s}\mathcal{L}\{t^3+9t^2+27t+27-4t-12+4\} = e^{-3s}\mathcal{L}\{t^3+9t^2+23t+19\} = e^{-3s}\left(\frac{3!}{s^4} + 9\cdot\frac{2!}{s^3} + 23\frac{1}{s^2} + \frac{19}{s}\right)$$

$$= e^{-3s}\left(\frac{6}{s^4} + \frac{18}{s^3} + \frac{23}{s^2} + \frac{19}{s}\right)$$

- $\mathcal{L}\{(-t^3-2)\, \underset{c=4}{u_4(t)}\} = -e^{-4s}\, \mathcal{L}\{(t+4)^3+2\} =$

$$= e^{-4s}\mathcal{L}\{t^3 + \underbrace{3\cdot 4}_{12}t^2 + \underbrace{3\cdot 4^2}_{48}t + 64+2\} =$$

$$= e^{-4s}\mathcal{L}\{t^3+12t^2+48t+66\} = e^{-4s}\left(\frac{3!}{s^4} + 12\cdot\frac{2!}{s^3} + 48\frac{1}{s^2} + \frac{66}{s}\right)$$

$$= e^{-4s}\left(\frac{6}{s^4} + \frac{24}{s^3} + \frac{48}{s^2} + \frac{66}{s}\right)$$

Combining all together we get

$$\boxed{\mathcal{L}\{f(t)\} = \frac{2}{s^2} - \frac{1}{s} + e^{-3s}\left(\frac{6}{s^4} + \frac{18}{s^3} + \frac{23}{s^2} + \frac{19}{s}\right) + e^{-4s}\left(\frac{6}{s^4} + \frac{24}{s^3} + \frac{48}{s^2} + \frac{66}{s}\right)}$$

Problem 2 Find the inverse Laplace transform of the function

$$\frac{e^{-\frac{3\pi}{2}s}\left(s^2+3s+4\right)}{(s+1)^2\left(s^2+6s+25\right)}$$

1) First find the inverse Laplace transform of

$$\frac{s^2+3s+4}{(s+1)^2\underbrace{(s^2+6s+25)}_{(s+3)^2+4^2}} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C(s+3)+4D}{(s+3)^2+4^2}$$

$$s^2+3s+4 = A(s+1)\left((s+3)^2+4^2\right) + B\left((s+3)^2+4^2\right) + \left(C(s+3)+4D\right)(s+1)^2$$

- To find B plug $s=-1$: $\underbrace{1-3+4}_{2} = B\cdot 20 \Rightarrow B = \frac{1}{10}$
- Plug $s=-3$: $\cancel{9}-\cancel{9}+4 = A(-3+1)\cdot 16 + \underbrace{\underbrace{B}_{\frac{1}{10}}\cdot 16}_{\frac{8}{5}} + 4D\cdot(-3+1)^2 \Leftrightarrow$

$$-32A + 16D = 4 - \frac{8}{5} = \frac{12}{5} \Leftrightarrow$$

$$8A - 4D = -\frac{3}{5}$$

- Plug $s=0$: $4 = A\cdot 25 + \underbrace{\frac{1}{10}\cdot 25}_{5/2} + 3C + 4D \Leftrightarrow$

$$25A + 3C + 4D = 4 - \frac{5}{2} = \frac{3}{2}$$

Compare coefficient of s^3:

$$0 = A + C$$

So, we have the following system for A, C, D:

$$\begin{cases} 8A - 4D = -\frac{3}{5} \\ 25A + 3C + 4D = \frac{3}{2} \\ C = -A \end{cases}$$

$$8A - 4D = -\frac{3}{5} \quad \text{Eq (1)}$$

$$25A - 3A + 4D = \frac{3}{2} \iff 22A + 4D = \frac{3}{2} \quad (\text{Eq } 2)$$

To eliminate D: (Eq 2) + (Eq 1):

$$\underbrace{(22+8)}_{30} A = \frac{3}{2} - \frac{3}{5} = \frac{9}{10} \Rightarrow$$

$$\Rightarrow \boxed{A = \frac{9}{30 \cdot 10} = \frac{3}{100}} \Rightarrow \boxed{C = -A = -\frac{3}{100}}$$

$$4D = 8A + \frac{3}{5} = 8 \cdot \frac{3}{100} + \frac{3}{5} = \frac{21}{25} \Rightarrow \boxed{D = \frac{21}{100}}$$

$$\Rightarrow \frac{s^2+3s+4}{(s+1)^2(s^2+6s+25)} = \frac{3}{100}\frac{1}{s+1} + \frac{1}{10}\frac{1}{(s+1)^2} - \frac{3}{100}\frac{s+3}{(s+3)^2+4^2} + \frac{21}{100}\frac{4}{(s+3)^2+4^2}$$

$$\Downarrow$$

$$\mathcal{L}^{-1}\left\{\frac{s^2+3s+4}{(s+1)^2(s^2+6s+25)}\right\} = \frac{3}{100}e^{-t} + \frac{1}{10}te^{-t} - \frac{3}{100}e^{-3t}\cos 4t + \frac{21}{100}e^{-3t}\sin 4t$$

$$\Downarrow \quad c = \frac{3\pi}{2}$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-\frac{3\pi}{2}s}(s^2+3s+4)}{(s+1)^2(s^2+6s+25)}\right\} = u_{\frac{3\pi}{2}}(t)\left(\frac{3}{100}e^{-\left(t-\frac{3\pi}{2}\right)} + \frac{1}{10}\left(t-\frac{3\pi}{2}\right)e^{-\left(t-\frac{3\pi}{2}\right)} - \frac{3}{100}e^{-3\left(t-\frac{3\pi}{2}\right)}\cos\left(4\left(t-\frac{3\pi}{2}\right)\right) + \frac{21}{100}e^{-3\left(t-\frac{3\pi}{2}\right)}\sin\left(4\left(t-\frac{3\pi}{2}\right)\right)\right)$$

$$= u_{\frac{3\pi}{2}}(t)\left(\frac{3}{100}e^{\frac{3\pi}{2}}e^{-t} + \frac{1}{10}e^{\frac{3\pi}{2}}te^{-t} - \frac{3\pi}{20}e^{\frac{3\pi}{2}}e^{-t} -\right.$$

$$\left. -\frac{3}{100}e^{\frac{9\pi}{2}}e^{-3t}\underbrace{\cos(4t-6\pi)}_{\cos 4t} + \frac{21}{100}e^{\frac{9\pi}{2}}e^{-3t}\underbrace{\sin(4t-6\pi)}_{\sin 4t}\right) =$$

$$= \boxed{u_{\frac{3\pi}{2}}(t)\left(\left(\frac{3}{100}-\frac{3\pi}{20}\right)e^{\frac{3\pi}{2}}e^{-t} + \frac{1}{10}e^{\frac{3\pi}{2}}te^{-t} - \frac{3}{100}e^{\frac{9\pi}{2}}e^{-3t}\cos 4t + \frac{21}{100}e^{\frac{9\pi}{2}}e^{-3t}\sin 4t\right.}$$

Problem 3 Find the solution of the initial value problem

$$y'' + 10y' + 29y = g(t),\ y(0) = -2,\ y'(0) = 1,\ \text{where}$$

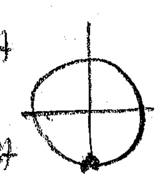
$$g(t) = \begin{cases} 4\cos 3t, & 0 \le t < \frac{5\pi}{2} \\ 2 + 5\sin 3t, & t \ge \frac{5\pi}{2} \end{cases}$$

Solution 1) $g(t) = 4\cos 3t + (2 + 5\sin 3t - 4\cos 3t)u_{\frac{5\pi}{2}}(t)$

$$\mathcal{L}\{4\cos 3t\} = 4\frac{s}{s^2+9}$$

$$\mathcal{L}\{(2+5\sin 3t - 4\cos 3t)\underbrace{u_{\frac{5\pi}{2}}(t)}_{c=\frac{5\pi}{2}}\} = e^{-\frac{5\pi}{2}s}\mathcal{L}\{2+5\sin 3(t+\tfrac{5\pi}{2}) - 4\cos 3(t+\tfrac{5\pi}{2})\} = e^{-\frac{5\pi}{2}s}\mathcal{L}\{2+5\sin(3t+\tfrac{15\pi}{2}) - 4\cos(3t+\tfrac{15\pi}{2})\}$$

$$\frac{15\pi}{2} + 3t = \underbrace{6\pi}_{3\cdot 2\pi} + \frac{3\pi}{2} + 3t$$



$$\frac{3\pi}{2}$$

$$\Rightarrow \sin\left(3t + \frac{15\pi}{2}\right) = \sin\left(3t + \frac{3\pi}{2}\right) = -\cos 3t$$

$$\cos\left(3t + \frac{15\pi}{2}\right) = \cos\left(3t + \frac{3\pi}{2}\right) = \sin 3t$$

$$= e^{-\frac{5\pi}{2}s}\mathcal{L}\{2 - 5\cos 3t - 4\sin 3t\} = e^{-\frac{5\pi}{2}s}\left(\frac{2}{s} - \frac{5s}{s^2+9} - \frac{12}{s^2+9}\right)$$

$\beta = 3$

2) Left handside of the differential equation

$29 \times \quad \mathcal{L}\{y\} = Y(s)$

$+ \quad 10 \times \quad \mathcal{L}\{y'(t)\} = sY(s) - y(0) = sY(s) - (-2) = sY(s) + 2$

$+ \quad \mathcal{L}\{y''(t)\} = s^2Y(s) - sy(0) - y'(0) = s^2Y(s) + 2s - 1$

$$\Rightarrow \quad (s^2+10s+29)Y(s) + 2s - 1 + 20 = (s^2+10s+29)Y(s) + 2s + 19 = \underbrace{\frac{4s}{s^2+9} + e^{-\frac{5\pi}{2}s}\left(\frac{2}{5} - \frac{5s}{s^2+9} - \frac{12}{s^2+9}\right)}$$

comparing with the right handside

$$Y(s) = -\frac{2s+19}{s^2+10s+29} + e^{-\frac{5\pi}{2}s}\left(\frac{1}{s^2+10s+29}\left(\frac{2}{5} - \frac{5s+12}{s^2+9}\right)\right) + \frac{4s}{(s^2+10s+29)(s^2+9)}$$

3) $\mathcal{L}^{-1}\{Y(s)\}$?

- $\mathcal{L}^{-1}\left\{\frac{2s+19}{\underbrace{s^2+10s+29}_{(s+5)^2+2^2}}\right\} = ?$

$\alpha = -5 \quad \beta = 2$

Rem: we could combine the 1st and the 2nd term but we do them separately

$$\frac{2s+19}{s^2+10s+29} = \frac{A(s+5)+2B}{s^2+10s+29} \Rightarrow 2s+19 = A(s+5)+2B$$

Plug $s=-5$: $-10+19 = 2B \Rightarrow B = \frac{9}{2}$

Compare coefficient of s:

$A = 2$

$$\Rightarrow \frac{2s+9}{s^2+10s+29} = 2\frac{s+5}{(s+5)^2+2^2} + \frac{9}{2}\frac{2}{(s+5)^2+2^2} \Rightarrow$$

$$\mathcal{L}^{-1}\left\{\frac{2s+9}{s^2+10s+29}\right\} = 2e^{-5t}\cos 2t + \frac{9}{2}e^{-5t}\sin 2t$$

b) $\mathcal{L}^{-1}\left\{e^{-\frac{5\pi}{2}s}\frac{2}{s(s^2+10s+29)}\right\}$?

First find $\mathcal{L}^{-1}\left\{\frac{2}{s(s^2+10s+29)}\right\}$

$$\frac{2}{s(\underbrace{s^2+10s+29}_{(s+5)^2+2^2})} = \frac{A}{s}+\frac{B(s+5)+2C}{(s+5)^2+2^2}$$

$$2 = A\left((s+5)^2+2^2\right)+\left(B(s+5)+2C\right)s$$

Find A by plugging $s=0$:

$$2 = 29A \Rightarrow \boxed{A=\frac{2}{29}}$$

Plug $s=-5$: $2 = 4A-10C \Rightarrow 5C = 2A-1 = \frac{4}{29}-1 = -\frac{25}{29} \Rightarrow \boxed{C=-\frac{5}{29}}$

Compare coefficient of s^2: $0 = A+B \Rightarrow \boxed{B=-\frac{2}{29}}$

$\Downarrow$

$$\frac{2}{s(s^2+10s+29)} = \frac{2}{29}\frac{1}{s} - \frac{2}{29}\frac{s+5}{(s+5)^2+2^2} - \frac{5}{29}\frac{2}{(s+5)^2+2^2} \Rightarrow$$

$$\mathcal{L}^{-1}\left\{\frac{2}{s(s^2+10s+29)}\right\} = \frac{2}{29} - \frac{2}{29}e^{-5t}\cos 2t - \frac{5}{29}e^{-5t}\sin 2t$$

$$\Rightarrow \mathcal{L}^{-1}\left\{e^{-\frac{5\pi}{2}s}\frac{2}{s(s^2+10s+29)}\right\} = u_{\frac{5\pi}{2}}(t)\left(\frac{2}{29}-\frac{2}{29}e^{-5(t-\frac{5\pi}{2})}\cos 2(t-\tfrac{5\pi}{2}) - \frac{5}{29}e^{-5(t-\frac{5\pi}{2})}\sin 2(t-\tfrac{5\pi}{2})\right) = u_{\frac{5\pi}{2}}(t)\left(\frac{2}{29}-\frac{2}{29}e^{\frac{25\pi}{2}}e^{-5t}\cos(2t-5\pi) - \frac{5}{29}e^{\frac{25\pi}{2}}e^{-5t}\sin(2t-5\pi)\right) = u_{\frac{5\pi}{2}}(t)\left(\frac{2}{29}+\frac{2}{29}e^{\frac{25\pi}{2}}e^{-5t}\cos 2t+\frac{5}{29}e^{\frac{25\pi}{2}}e^{-5t}\sin 2t\right)$$

$2t-5\pi = 2t-\pi-4\pi$

$\Rightarrow \cos(2t-5\pi) = -\cos 2t$

$\sin(2t-5\pi) = -\sin 2t$

c) $\mathcal{L}^{-1}\left\{ e^{-\frac{5\pi}{2}s} \frac{5s+12}{(s^2+10s+29)(s^2+9)} \right\}$?

First find $\mathcal{L}^{-1}\left\{\frac{5s+12}{(s^2+10s+29)(s^2+9)}\right\}$

$$\frac{5s+12}{\underbrace{(s^2+10s+29)}_{(s+5)^2+2^2}(s^2+9)} = \frac{A(s+5)+2B}{(s+5)^2+2^2} + \frac{Cs+3D}{s^2+9}$$

$$5s+12 = \left(A(s+5)+2B\right)(s^2+9) + (Cs+3D)\left(\overbrace{(s+5)^2+2^2}^{s^2+10s+29}\right)$$

$s=0$: $12 = (5A+2B)\cdot 9 + 3D\cdot 29 = 45A+18B+87D$

$s=-5$: $-13 = 2B\cdot 34 + (-5C+3D)\cdot 4 = 68B-20C+12D$

coefficient of s^3: $0 = A+C \Rightarrow C=-A$

Coefficient of s^2: $0 = 5A+2B+\underbrace{10C}_{-10A}+3D$

$$\Rightarrow \begin{cases} 45A+18B+87D = 12 \\ 20A+68B+12D = -13 \\ -5A+2B+3D = 0 \end{cases}$$

Augmented matrix: $\left(\begin{array}{ccc|c} 45 & 18 & 87 & 12 \\ 20 & 68 & 12 & -13 \\ -5 & 2 & 3 & 0 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c} -5 & 2 & 3 & 0 \\ 20 & 68 & 12 & -13 \\ 45 & 18 & 87 & 12 \end{array}\right) \sim$

$R_2 \to R_2+4R_1$, $R_3 \to R_3+9R_1$: $\left(\begin{array}{ccc|c} -5 & 2 & 3 & 0 \\ 0 & 76 & 24 & -13 \\ 0 & 36 & 114 & 12 \end{array}\right) \xrightarrow{R_2 \to R_2-2R_3} \left(\begin{array}{ccc|c} -10 & 2 & 3 & 0 \\ 0 & 4 & -204 & -37 \\ 0 & 36 & 114 & 12 \end{array}\right) \xrightarrow{R_3 \to R_3-9R_2}$

$R_3 \to R_3-9R_2$: $\sim \left(\begin{array}{ccc|c} -5 & 2 & 3 & 0 \\ 0 & 4 & -204 & -37 \\ 0 & 0 & 1950 & 345 \end{array}\right) \Rightarrow$

$-5A - \frac{59}{130} + \frac{69}{130} = 0 \Rightarrow 5A = \frac{1}{13} \Rightarrow A = \boxed{\frac{1}{65}}$

$4B - \frac{204\cdot 23}{130} = -37 \Rightarrow B = \frac{1}{4}\left(-37+\frac{204\cdot 23}{130}\right) = \boxed{-\frac{59}{260}}$

$1950D = 345 \Rightarrow D = \frac{345}{1950} = \boxed{\frac{23}{130}}$

$\boxed{C = -A = -\frac{1}{65}}$

$$\Rightarrow \frac{5s+12}{(s^2+10s+29)(s^2+9)} = \frac{1}{65}\frac{s+5}{(s+5)^2+2^2} - \frac{59}{260}\frac{2}{(s+5)^2+2^2} - \frac{1}{65}\frac{s}{s^2+9} + \frac{23}{130}\cdot\frac{3}{s^2+9}$$

$$\Downarrow$$

$$\mathcal{L}^{-1}\left\{\frac{5s+12}{(s^2+10s+29)(s^2+9)}\right\} = \frac{1}{65}e^{-5t}\cos 2t - \frac{59}{260}e^{-5t}\sin 2t - \frac{1}{65}\cos 3t + \frac{23}{130}\sin 3t$$

$$\Downarrow$$

$$\mathcal{L}^{-1}\left\{e^{-\frac{5\pi}{2}s}\frac{5s+12}{(s^2+10s+29)(s^2+9)}\right\} = u_{\frac{5\pi}{2}}(t)\left(\frac{1}{65}e^{-5(t-\frac{5\pi}{2})}\cos\big(\underbrace{2(t-\tfrac{5\pi}{2})}_{2t-5\pi}\big) - \right.$$

$$\left. - \frac{59}{260}e^{-5(t-\frac{5\pi}{2})}\sin\underbrace{2(t-\tfrac{5\pi}{2})}_{2t-5\pi}\Big) - \frac{1}{65}\cos\underbrace{3(t-\tfrac{5\pi}{2})}_{3t-\frac{15\pi}{2}} + \frac{23}{130}\sin\underbrace{3(t-\tfrac{5\pi}{2})}_{3t-\frac{15\pi}{2}}\Big)\right) -$$

$$= u_{\frac{5\pi}{2}}(t)\left(-\frac{1}{65}e^{\frac{25\pi}{2}}e^{-5t}\cos 2t + \frac{59}{260}e^{\frac{25\pi}{2}}e^{-5t}\sin 2t + \frac{1}{65}\sin 3t + \frac{23}{130}\cos 3t\right)$$

As we already discussed before (see end of page 6 and end of page 4) ①

$\cos(2t-5\pi) = -\cos 2t$ $\qquad$ $\sin\left(3t-\frac{15\pi}{2}\right) = \sin\left(3t+\frac{3\pi}{2}\right) = \cos 3t$

$\sin(2t-5\pi) = -\sin 2t$ $\qquad$ $\cos\left(3t-\frac{15\pi}{2}\right) = \cos\left(3t-\frac{3\pi}{2}\right) = -\sin 3t$

$-\frac{15\pi}{2} = -6\pi - \frac{3\pi}{2}$

d) $\mathcal{L}^{-1}\left(\frac{4s}{(s^2+10s+29)(s^2+9)}\right)$?

$$\frac{4s}{(\underbrace{s^2+10s+29}_{(s+5)^2+2^2})(s^2+9)} = \frac{A(s+5)+2B}{(s+5)^2+2^2} + \frac{Cs+3D}{s^2+9}$$

$$4s = \big(A(s+5)+2B\big)(s^2+9) + (Cs+3D)\big((s+5)^2+2^2\big)$$

Proceed similarly to item c) (see page 7 → the right-hand side of the system for A, B, D here will be the same)

- $s=0$: $\quad 0 = 45A + 18D + 87D$
- $s=-5$: $\quad -20 = 68B - 20C + 12D$
- coefficient of s^3: $\quad 0 = A + C \Rightarrow C = -A$

Coefficient of s^2: $0 = 5A + 2B + \underbrace{10C}_{-10A} + 3D = -5A + 2B + 3D$

$\Downarrow$

$45A + 18B + 87D = 0$

$20A + 68B + 12D = -20$

$-5A + 2B + 3D = 0$

Augmented matrix: $\left(\begin{array}{ccc|c} 45 & 18 & 87 & 0 \\ 20 & 68 & 12 & -20 \\ -5 & 2 & 3 & 0 \end{array}\right) \sim$ as in page 7 just the last column is different

$\sim \left(\begin{array}{ccc|c} -5 & 2 & 3 & 0 \\ 0 & 76 & 24 & -20 \\ 0 & 36 & 114 & 0 \end{array}\right) \overset{R_2 \to R_2 - 2R_3}{\sim} \left(\begin{array}{ccc|c} -5 & 2 & 3 & 0 \\ 0 & 4 & -204 & -20 \\ 0 & 36 & 114 & 0 \end{array}\right) \begin{array}{l} R_2 \to \frac{1}{4}R_2 \\ \sim \\ R_3 \to \frac{1}{2}R_3 \end{array}$

$\left(\begin{array}{ccc|c} -5 & 2 & 3 & 0 \\ 0 & 1 & -51 & -5 \\ 0 & 18 & 57 & 0 \end{array}\right) \overset{R_3 \to R_3 - 18R_2}{\sim} \left(\begin{array}{ccc|c} -5 & 2 & 3 & 0 \\ 0 & 1 & -51 & -5 \\ 0 & 0 & 975 & 90 \end{array}\right) \Rightarrow$

$975D = 90 \Rightarrow D = \frac{90}{975} = \frac{6}{65}$

$B - 51 \cdot \frac{6}{65} = -5 \Rightarrow B = -\frac{19}{65}$

$-5A - \frac{38}{65} + \frac{18}{65} = 0 \Rightarrow A = -\frac{4}{65}$

$C = -A = \frac{4}{65}$

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{4s}{(s^2+10s+29)(s^2+9)}\right\} = \mathcal{L}^{-1}\left\{-\frac{4}{65}\frac{s+5}{(s+5)^2+2^2} - \frac{19}{65}\cdot\frac{2}{(s+5)^2+2^2} + \frac{4}{65}\frac{s}{s^2+9} + \frac{6}{65}\frac{3}{s^2+9}\right\}$$

$$= -\frac{4}{65}e^{-5t}\cos 2t - \frac{19}{65}e^{-5t}\sin 2t + \frac{4}{65}\cos 3t + \frac{6}{65}\sin 3t$$

Combining (a), (d), (b), (c)

$$y(t) = -2e^{-5t}\cos 2t - \frac{9}{2}e^{-5t}\sin 2t - \frac{4}{65}e^{-5t}\cos 2t - \frac{19}{65}e^{-5t}\sin 2t + \frac{4}{65}\cos 3t + \frac{6}{65}\sin 3t + u_{\frac{5\pi}{2}}(t)\left(\frac{2}{29} + \frac{2}{29}e^{\frac{25\pi}{2}}e^{-5t}\cos 2t + \frac{5}{29}e^{\frac{25\pi}{2}}e^{-5t}\sin 2t + \frac{1}{65}e^{\frac{25\pi}{2}}e^{-5t}\cos 2t - \frac{59}{260}e^{\frac{25\pi}{2}}e^{-5t}\sin 2t - \frac{1}{65}\sin 3t - \frac{23}{130}\cos 3t\right) =$$

y (ex 10)

$$= -\frac{134}{65} e^{-5t}\cos 2t - \frac{623}{130} e^{-5t}\sin 2t + \frac{4}{65}\cos 3t + \frac{6}{65}\sin 3t + u_{\frac{5\pi}{2}}(t)\Big(\frac{2}{29} +$$

$$+ \frac{159}{1885} e^{\frac{25\pi}{2}} e^{-5t}\cos 2t + \frac{411}{7540} e^{\frac{25\pi}{2}} e^{-5t}\sin 2t - \frac{1}{65}\sin 3t - \frac{23}{130}\cos 3t\Big)$$