Homework Assignment 14 in Differential Equations, MATH308-SPRING 2015

due April 17, 2015 *Section covered 7.6*

1. Given the following system of linear differential equations:

$$\begin{cases} x_1' &= -2x_1 - x_2 \\ x_2' &= 16x_1 - 2x_2 \end{cases} \tag{1}$$

   (a) Find the general solution of the system (1).

   (b) If $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ is a solution of (1), what is the limit of $x(t)$ as $t \to +\infty$. Does this limit depend on initial conditions?

   (c) Find the solution of the system (1) satisfying the initial conditions: $x_1(0) = -3, \quad x_2(0) = 2$.

2. Given the following system of linear differential equations:

$$\begin{cases} x_1' &= -3x_2 + 6x_3 \\ x_2' &= 4x_1 + 5x_2 + 4x_3 \\ x_3' &= x_1 - 7x_2 - 5x_3 \end{cases} \tag{2}$$

   (a) It is known that -6 is an eigenvalue of the corresponding matrix. Find the general solution of the system (2).

   (b) Find the solution of the the system (2) satisfying the initial condition $\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$

   (c) Find all $\alpha_1, \alpha_2, \alpha_3$ such that if $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$ is the solution of the system (2) with initial

   condition $x(0) = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$ then $x(t) \to 0$ as $t \to +\infty$.

   (d) (**bonus 15 pts**) Find all $\beta_1, \beta_2, \beta_3$ such that if $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$ is the solution of the system

   (2) with initial condition $x(0) = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$ then $x(t) \to 0$ as $t \to -\infty$.