Homework Assignment 15 in Differential Equations, MATH308-SPRING 2015

due April 22, 2015 *Sections covered: end of 7.5 (the case when there are repeated eigenvalues and a basis of eigenvectors)& 7.8 (when there are repeated eigenvalues but no basis of eigenvectors)*

1. Find a fundamental matrix and the general solution of the following system of linear differential equations:
$$\begin{cases} x_1' &= 5x_1 - 2x_2 + 3x_3 \\ x_2' &= -2x_1 + 2x_2 + 6x_3 \\ x_3' &= x_1 + 2x_2 + 3x_3 \end{cases}$$

2. Given the following system of linear differential equations:
$$\begin{cases} x_1' &= -13x_1 + 25x_2 \\ x_2' &= -9x_1 + 17x_2 \end{cases} \tag{1}$$

   (a) Find the general solution of the system (1).

   (b) If $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ is a solution of (1), what is the limit of $x(t)$ as $t \to -\infty$. Does this limit depend on initial conditions?

   (c) Find the solution of the system (1) satisfying the initial conditions: $x_1(0) = -4, \quad x_2(0) = 3$.

3. Find a fundamental matrix and the general solution of the following system of linear differential equations:
$$\begin{cases} x_1' &= x_2 + 2x_3 \\ x_2' &= 3x_1 + 2x_2 + 2x_3 \\ x_3' &= -2x_1 - 2x_2 - 3x_3 \end{cases}$$