Homework Assignment 17 in Differential Equations, MATH308-Spring 2015

This Homework is not for submission but it is highly recommended to solve it (the topics covered in it will be a part of the Final Exam)

Topics covered : *The Phase Plane: linear systems (section 9.1) and locally linear system (section 9.3): types of crticial (equillibrium) points and stability; sketch of the phase portrait for planar linear systems.*

1. For each of the following systems

   i) Find and *classify* the critical (equilibrium) point and determine whether it is stable, asymptotically stable, or unstable;

   ii) *Sketch* the phase portrait of the system (indicating *direction of motion* along trajectories by arrows and the direction of eigenvectors, if relevant):

   (a) $\begin{cases} x_1' &= 5x_1 + 6x_2 \\ x_2' &= -5x_1 - 8x_2, \end{cases}$ (b) $\begin{cases} x_1' &= 3x_1 + 5x_2 \\ x_2' &= -4x_1 - 5x_2, \end{cases}$

   (c) $\begin{cases} x_1' &= -6x_1 + 3x_2 + 3 \\ x_2' &= x_1 - 4x_2 + 10, \end{cases}$ (d) $\begin{cases} x_1' &= -2x_1 + x_2 - 6 \\ x_2' &= -x_1 - 4x_2 + 15. \end{cases}$

2. For each of the following systems

   i) Determine all critical points;

   ii) Find the corresponding linear system near each critical point;

   iii) Find the eigenvalues of each linear systems obtained in the previous item and determine the type of the critical point of these linear systems. On the base of this information what conclusions (if any) can be given on the stability properties of the corresponding critical points (whether they are stable, asymptotically stable, or unstable)?

   (a) $\begin{cases} x' &= 16 - xy \\ y' &= x - y^3, \end{cases}$ (b) $\begin{cases} x' &= (2 + x)(y - x) \\ y' &= (4 - x)(x + y). \end{cases}$