

-14-

For example,

1) If  $\det A < 0 \Rightarrow$  it is a saddle point  
(because then  $\lambda_1, \lambda_2 < 0$ )

2) If discriminant  $> 0$  &  $\det A > 0 \Rightarrow$  it is a node  
(because  $\lambda_1, \lambda_2$  are real and of the same sign)

If in addition to this  $\text{tr} A < 0 \Rightarrow$  node sink

(because then  $\lambda_1 + \lambda_2 < 0 \Rightarrow$  both of them are negative)

and if  $\text{tr} A > 0 \Rightarrow$  node source

(because then  $\lambda_1 + \lambda_2 > 0 \Rightarrow$  both of them are positive)

3) If  $\Delta < 0 \Rightarrow$  complex roots, then

$\text{tr} A < 0 \Rightarrow$  spiral sink

$\text{tr} A = 0 \Rightarrow$  center

$\text{tr} A > 0 \Rightarrow$  spiral source

4) If  $\Delta = 0 \Rightarrow$  repeated eigenvalue

If  $A = \lambda I \Rightarrow$  proper node

otherwise  $\Rightarrow$  improper node

If in addition  $\text{tr} A < 0 \rightarrow$  stable

$\text{tr} A > 0 \rightarrow$  unstable

See also

Figure 9.1.9

page 497

of the textbook  
nicely summarizing this

Rough sketch of the phase portrait for problem 21

