

Homework Assignment #1

FALL 2016 - MATH308

due September 5, 2016 at the beginning of class

Topics covered : equations $y' = ay + b$, where a and b are constant, and separable equations (corresponds to sections 1.2, 2.2 in the textbook). *You do not need to use calculator for this assignment.*

1. Assume that the velocity v of the falling object satisfies the following differential equation:

$$v'(t) = 19.6 - \frac{v(t)}{a} \quad (1)$$

where a is a positive constant.

- (a) Find a number v_e such that $v(t) \equiv v_e$ is a solution of equation (1) (in other words find the equilibrium solution of (1)).
 - (b) Solve the equation (1) with initial condition $v(0) = 0$. What is the limit of this solution when $t \rightarrow +\infty$? How this limiting velocity is related to your answer in the item (a)?
 - (c) Find the time that must elapse for the object to reach 75% of the limiting velocity found in the item (b).
 - (d) How far does the object fall in the time found in the item (c).
2. Solve the following differential equations (find the general solutions):
 - (a) Find the general solution of $y' = 1 - t + y^2 - ty^2$;
 - (b) Find the solution of the initial value problem $(1 + t^2)^{1/2} dy - ty^3(1 + t^2)^{-1/2} dt = 0$, $y(0) = 1$. Express y in terms of t .