Homework Assignment #2 Fall 2015 - MATH308-505/506

due January 30, 2015 at the beginning of class

Topics covered : direction field and qualitative analysis of autonomous equations on the line (corresponds to sections 1.1 and 2.5).

1. Given the differential equation:

$$y' = 3 - 2y \tag{1}$$

   (a) Find all equilibrium solutions.

   (b) Sketch a direction field.

   (c) Given any initial condition $y(0) = y_0$ describe the behavior of the corresponding solution when $t \to -\infty$ and the behavior of the corresponding solutions when $t \to +\infty$.

   (d) Solve the equation (1) analytically for each initial value $y(0) = y_0$ and justify your answer in the item (c) analyzing the obtained solution.

2. Given the differential equation:

$$y' = y^2 - 3y + 2 \tag{2}$$

   (a) Find all equilibrium points.

   (b) Sketch a direction field.

   (c) Based on the sketch of the direction field from the item (b) answer the following questions:

      i. Let $y(t)$ be the solution of equation (2) satisfying the initial condition $y(0) = \frac{4}{3}$. Find the limit of $y(t)$ when $t \to +\infty$ and the limit of $y(t)$ when $t \to -\infty$ (for this you do not need to find $y(t)$ explicitly).

      ii. Find all y_0 such that the solution of the equation (2) with the initial condition $y(0) = y_0$ has the same limit at $+\infty$ as the solution from the item (c)i.

      iii. Let $y(t)$ be the solution of equation (2) with $y(0) = 3$. Decide wether $y(t)$ is monotonically decreasing or increasing and find to what value it approaches when t increases (the value might be infinite).

   (d) (*bonus* - 20 points) Find the solution of the equation (2) with $y(0) = 3$ explicitly. Determine the interval in which this solution is defined.