Homework Assignment #2 FAll 2016 - MATH308

due September 12, 2016 at the beginning of class

Topics covered : method of integrating factor (sections 2.1); direction field and qualitative analysis of autonomous equations on the line (corresponds to sections 1.1 and 2.5) and a bonus question on equation that can be reduced to separable by an appropriate substitution.

1. (a) Solve the initial value problem
$$y' = -y\cot t + \cos t, \quad y\left(\frac{\pi}{2}\right) = 4.$$
   (b) Find the general solution of the differential equation
$$(1+t^2)y' + ty = (1+t^2)^{5/2}.$$
   (*Hint:* Divide both sides of the equation to $1+t^2$.)

2. Show that every solution of the equation $y'(t) + ay = be^{-ct}$, where a and c are positive constants and b is any real number, approaches zero as t approaches $+\infty$.

3. Given the differential equation:
$$y' = y^2 - 2y - 3 \tag{1}$$
   (a) Find all equilibrium points.
   (b) Sketch a direction field.
   (c) Based on the sketch of the direction field from the item (b) answer the following questions:
      i. Let $y(t)$ be the solution of equation (1) satisfying the initial condition $y(0) = 2$. Find the limit of $y(t)$ when $t \to +\infty$ and the limit of $y(t)$ when $t \to -\infty$ (for this you do not need to find $y(t)$ explicitly).
      ii. Find all y_0 such that the solution of the equation (1) with the initial condition $y(0) = y_0$ has the same limit at $-\infty$ as the solution from the item (c)i.
      iii. Let $y(t)$ be the solution of equation (1) with $y(0) = 4$. Decide wether $y(t)$ is monotonically decreasing or increasing and find to what value it approaches when t increases (the value might be infinite).
   (d) Find the solution of the equation (1) with $y(0) = 4$ explicitly. Determine the interval in which this solution is defined.

4. (*bonus* - 30 points) Before attempting this problem review the lecture notes from August 31, when we discussed the equation of the type $y' = f(\frac{y}{x})$ (so-called, homogeneous equations) and $y' = f(ax+by+c)$: the main idea here is to make an appropriate substitution to obtain a separable equation: $u(x) = \frac{y(x)}{x}$ in the first case and $u(x) = ax + by(x) + c$ in the second case. Then find the general solution of the following equations:
   (a) $y' = \dfrac{y-x}{x+y}$;
   (b) $y' = (3x+2y-1)^2$.