Homework Assignment #2 Spring 2017 - MATH308, Regular section

due February 2, 2017 at the beginning of class

Topics covered : method of integrating factor (sections 2.1); mixing model (section 2.3), exact equations and integrating factor (section 2.6).

1. (a) Find the general solution of the differential equation

$$(t^2-1)y' = ty + 2t(t^2-1), \quad |t|>1.$$

(*Hint:* Divide both sides of the equation by t^2-1.)

(b) Solve the initial value problem

$$xy' + 3y = \cos x, \quad y(\pi) = \frac{2}{\pi^3}.$$

2. A tank contains 200 gal of water and 60 oz of salt. Water containing a salt concentration of $(0.2 + 0.4\sin 4t)$ oz/gal flow into the tank at the rate of 10 gal/min, and the mixture in the tank flows out at the same rate. Let $Q(t)$ be the amount of salt in the tank at time t.

(a) Write the initial value problem for $Q(t)$, i.e. the differential equation for $Q(t)$ and the initial condition for $Q(0)$;

(b) Find $Q(t)$ at any time moment.

3. Check if the following equation is exact and if yes, solve the given initial-value problem:

$$y(\cos 2x)e^{xy} - 2(\sin 2x)e^{xy} + 2x = (3 - x(\cos 2x)e^{xy})\frac{dy}{dx}, \quad y(0) = 0.$$

(*Hint:* Move the right-hand side to the left.)

4. Find the value of parameter a for which the differential equation

$$(x + ye^{2xy})dx + axe^{2xy}dy = 0$$

is exact, and then find the general solution in the case of this value of a .

5. For the differential equation

$$x + e^y + \left(\frac{x^2}{2} + 2xe^y\right)\frac{dy}{dx} = 0$$

find the integrating factor depending on y only to make it exact and then solve the equation.