Homework Assignment #2 Spring 2017 - MATH308, Honors section

due February 2, 2017 at the beginning of class

Topics covered : method of integrating factor (sections 2.1); Bernoulli equation (extra-notes); mixing model (section 2.3), exact equations and integrating factor (section 2.6).

1. (a) Find the general solution of the differential equation
$$(t^2-1)y' = ty + 2t(t^2-1), \quad |t|>1.$$
(*Hint:* Divide both sides of the equation by t^2-1.)

   (b) Solve the initial value problem
$$xy' + 3y = \cos x, \quad y(\pi) = \frac{2}{\pi^3}.$$

2. A pond contains 200 gal of water and 60 oz of salt. Water containing a salt concentration of $(0.2 + 0.4\sin 4t)$ oz/gal flow into the pond at the rate of 10 gal/min, and the mixture in the pond flows out at the same rate. Let $Q(t)$ be the amount of salt in the pond at time t.

   (a) Write the initial value problem for $Q(t)$, i.e. the differential equation for $Q(t)$ and the initial condition for $Q(0)$;

   (b) Find $Q(t)$ at any time moment.

   (c) Assume that we make the following change in the model: instead of the condition that the mixture in the pond flows out at the same rate as it flows in, assume that it flows out with the rate 9 gal/min (ignoring the fact that the volume of water in the pond will increase unboundedly). Write the initial value problem for $Q(t)$, i.e. the differential equation for $Q(t)$ and the initial condition for $Q(0)$ (do not solve it).

3. This exercise is devoted to Bernoulli equations, which are equations of the type
$$y' + p(t)y = q(t)y^\alpha, \quad \alpha \neq 0, 1$$
This is a nonlinear differential equation but by making a change of the dependent variable $v = y^{1-\alpha}$ this equation can be converted to a linear one and the method of integrating factor can be applied (the extra notes on this type of equations will be posted soon). Using the method of the previous item find the general solution of the equation
$$ty' - 4y = 2t^2\sqrt{y}, \quad t>0.$$

4. Check if the following equation is exact and if yes, solve the given initial-value problem:
$$y(\cos 2x)e^{xy} - 2(\sin 2x)e^{xy} + 2x = (3 - x(\cos 2x)e^{xy})\frac{dy}{dx}, \quad y(0)=0.$$
(*Hint:* Move the right-hand side to the left.)

5. Find the value of parameter a for which the differential equation
$$(x + ye^{2xy})dx + axe^{2xy}dy = 0$$
is exact, and then find the general solution in the case of this value of a .

6. (a) For the differential equation
$$x + e^y + \left(\frac{x^2}{2} + 2xe^y\right)\frac{dy}{dx} = 0$$
find the integrating factor depending on y only to make it exact and then solve the equation.

   (b) The differential equation
$$e^t \sec y - \tan y + \frac{dy}{dt} = 0$$
has an integrating factor of the form $e^{at}\cos y$ for some constant a. Find a, and then solve the differential equation.