

due June 4, 2012 at the beginning of class

Topics covered : separable equations (corresponds to sections 2.2); direction field and qualitative analysis of autonomous equations on the line (corresponds to sections 1.1 and 2.5).

1. Solve the following initial value problem:

$$y' - xy^2 = xy, \quad y(0) = 1.$$

2. Given the differential equation:

$$y' = y^2 - 4 \tag{1}$$

- (a) Find all equilibrium points.
- (b) Sketch a direction field.
- (c) Based on the sketch of the direction field from the item (b) answer the following questions:
 - i. Let $y(t)$ be the solution of equation (1) satisfying the initial condition $y(0) = -\frac{1}{3}$. Find the limit of $y(t)$ when $t \rightarrow +\infty$ and the limit of $y(t)$ when $t \rightarrow -\infty$ (for this you do not need to find $y(t)$ explicitly).
 - ii. Find all y_0 such that the solution of the equation (1) with the initial condition $y(0) = y_0$ has the same limit at $+\infty$ as the solution from the item (c)i.
 - iii. Let $y(t)$ be the solution of equation (1) with $y(0) = 3$. Decide whether $y(t)$ is monotonically decreasing or increasing and find to what value it approaches when t increases (the value might be infinite).
- (d) (*bonus* - 10 points) Find the solution of the equation (1) with $y(0) = 3$ explicitly. Determine the interval in which this solution is defined.