Homework Assignment 3 in Differential Equations, MATH308-FALL 2016

due September 19, 2016

Topics covered : *modeling with first order equation: mixing problem (section 2.3), existence and uniqueness of solutions for linear equations (section 2.4); exact equations (sections 2.6), bonus question- Bernoulli equation*

1. A tank contains 300 gal of water and 40 oz of salt. Water containing a salt concentration of $(0.5 - 0.2\sin 2t)$ oz/gal flow into the tank at the rate of 5 gal/min, and the mixture in the tank flows out at the same rate. Let $Q(t)$ be the amount of salt in the tank at time t.
   (a) Write the differential equation for $Q(t)$. What initial condition does $Q(t)$ satisfy?
   (b) Find $Q(t)$ at any time moment;
2. Consider the differential equation
$$(4t^2 + 5t - 6)y' + \ln|2t^2 + 9t - 5|y = e^{t^3} + \sin(t^2).$$
   In each of the following five items determine (without solving the equation) an interval in which the solution with given initial condition is certain to exist if the initial condition is
   (a) $y(1/4) = -3$, (b) $y(5/8) = -50$, (c) $y(-10) = 0$, (d) $y(1) = -100$, (e) $y(-1) = 1$.
3. Find the general solution of the given differential equation:
$$y\sec^2 x + \sec x \tan x + (2y + \tan x)\frac{dy}{dx} = 0$$
   and then the solution satisfying the initial condition $y(0) = -2$.
4. Find the value of parameter a for which the differential equation
$$(e^{ax+y} + 3x^2y^2)dx = -(2yx^3 + e^{ax+y})dy$$
   is exact, and then find the general solution in the case of this value of a .
5. Find all functions $f(x)$ such that the differential equation
$$y^2 \sin x + yf(x)\frac{dy}{dx} = 0$$
   is exact. Then find the general solution of the differential equations for all these $f(x)$.
6. (*bonus* - 30 points) Sometimes it is possible to solve a nonlinear differential equation by making a change of the dependent variables that converts it into a linear equation. The most important such equation is of the
$$y' + p(t)y = q(t)y^n, \quad n \neq 0, 1$$
   and it is called Bernoulli equation after Jakob Bernoulli (note that for $n = 0, 1$ such equation is already linear).
   (a) Show that the substitution $v = y^{1-n}$ reduce the Bernoulli equation to a linear equation.
   (b) Using the method of the previous item find the general solution of the equation
$$t^2y' + 2ty - y^3 = 0, \quad t > 0.$$