Homework Assignment 3 in Differential Equations, MATH308–Spring 2015

due February 4, 2015

Topics covered : *method of integrating factor (sections 2.1), modeling with first order equation (section 2.3).*

1. Find the general solution of the differential equation

$$y' + 3y = 4e^{-2t},$$

and determine how the solutions behave as $t \to +\infty$.

2. (a) Solve the initial value problem

$$y' - \frac{2y}{t+1} = e^{-3t}(t+1)^2, \quad y(0) = a. \tag{1}$$

(b) How do the solutions of (1) behave as t goes to $+\infty$? Show that this behavior depend on the choice of the initial value a and find the value a_0 for which the transition from one type of behavior to another occurs;

(c) Describe the behavior of the solution of (1) corresponding to the initial condition $y(0) = a_0$, where a_0 is as in the previous item.

3. A tank contains 200 gal of water and 25 oz of salt. Water containing a salt concentration of $(1 + \frac{1}{2} \sin t)$ oz/gal flow into the tank at the rate of 4 gal/min, and the mixture in the tank flows out at the same rate. Let $Q(t)$ be the amount of salt in the tank at time t.

(a) Write the differential equation for Q(t). What initial condition does $Q(t)$ satisfy?

(b) Find $Q(t)$ at any time moment;