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Homework #4 Solution of Bonus question for honors part

Problem 1 f) Existence of c

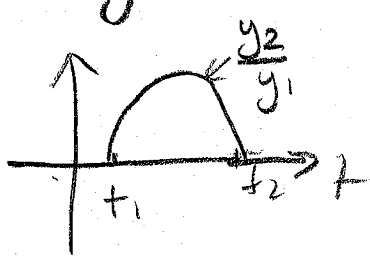
Assume by contradiction that there is no such c, i.e. $y_1(t) \neq 0$ for any t on (t_1, t_2). Then the function $\frac{y_2(t)}{y_1(t)}$ is differentiable on (t_1, t_2). Note that

$$\frac{d}{dt}\left(\frac{y_2(t)}{y_1(t)}\right) = \frac{y_2'(t)y_1(t) - y_1'(t)y_2(t)}{y_1^2(t)} =$$

$$= \frac{W(y_1, y_2)(t)}{y_1^2(t)}$$

Since y_1 & y_2 form a fundamental set of solutions $W(y_1, y_2)(t)$ is never zero $\Rightarrow \frac{d}{dt}\left(\frac{y_2(t)}{y_1(t)}\right) \neq 0$ for any $t \in (t_1, t_2)$. On the other hand

$\frac{y_2(t_1)}{y_2(t_1)} = 0$ and $\frac{y_2(t_2)}{y_2(t_2)} = 0 \Rightarrow$ there exist $a \in (t_1, t_2)$ s.t. $\left.\frac{d}{dt}\left(\frac{y_2}{y_1}\right)\right|_{t=a} = 0$



(In calculus this is called Rolle's theorem) $\Rightarrow$ contradiction

The proof of existence of c is completed.

Page 2 Uniqueness

Assume again by contradiction that there exist two c_1 & c_2 in (t_1, t_2), $c_1 < c_2$ such that $y_1(c_1) = y_1(c_2) = 0$. Then we can swap the role of y_1 and y_2 and do apply the existence argument we used before to conclude that there exist $c_3 \in (c_1, c_2)$ s.t $y_2(c_3) = 0$. This contradicts the assumption that t_1 and t_2 are two consecutive zeros of y_2. The proof of uniqueness is completed.