Homework Assignment 4 in Differential Equations, MATH308-SUMMER 2012

due to June 8, 2012

Topics covered : *modeling with first order equation; existence and uniqueness of solutions for linear equations (corresponds to sections 2.3, 2.4 in the textbook).*

1. A tank originally contains 240 gal of fresh water. Then water containing $\frac{1}{6}$ lb of salt per gallon is poured into the tank at a rate of 12 gal/min, and the mixture is allowed to leave at the same rate. After 20 min the process is stopped. Then the water containing $\frac{1}{2}$ lb of salt per gallon is poured into the tank at a rate of 6 gal/min, with the mixture again leaving at the same rate. Find the amount of salt in the tank at the end of an additional 10 min.

2. Consider the differential equation

$$(t^2 + 5t + 6))y' + (t - 1)y = \sin t.$$

In each of the following three items determine (without solving the equation) an interval in which the solution with given initial condition is certain to exist if the initial condition is

(a) $y(-4) = 99$ (b) $y(-5/2) = -99$ (c) $y(-1) = 0$.