Homework Assignment 5 in MATH 308-Fall 2016
due October 5, 2016

Topics covered : *linear homogeneous equations of second order with constant coefficient: the cases of real roots (an additional exercise), repeated roots (section 3.4), complex roots (section 3.3)*, mechanical vibratons (section 3.7);use that the gravitational acceleration $g = 32\frac{ft}{s^2}$

1. Consider the equation $y'' + (\alpha + 1)y' + (\alpha - 2)(1 - 2\alpha)y = 0$, where α is a parameter.
   (a) Determine the values of α , if any for which all solutions tend to zero as $t \to \infty$.
   (b) Determine the values of α , if any for which all nonzero solutions become unbounded as $t \to \infty$.

   Hint: It is more efficient here to use the Vieta theorem for roots of quadratic equation instead of the quadratic formula

2. Consider the differential equation $16y'' + 24y' + 9y = 0$.
   (a) Find the solution satisfying the initial conditions $y(0) = -1$, $y'(0) = \alpha$;
   (b) For the solutions obtained in the previous item find the values of α , if any, for which the solutions tends to $+\infty$ as $t \to -\infty$ and the values of α, if any, for which the solutions tend to $-\infty$ as $t \to -\infty$.

3. (a) Write the given expressions in the form $a + ib$:
   (i) $(3 - 2i)(10 + 5i)$; (ii) $\dfrac{3 - 4i}{2 - 5i}$ (Hint for (ii): multiply both numerator and denominator by complex conjugate of the denominator)
   (b) Use Euler's formula to write the given expression in the form $a + ib$:
   (i) $e^{-2-\frac{11\pi}{6}i}$; (ii) $(1 - i)^9$.

4. Consider the differential equation $9y'' + 12y' + 20y = 0$.
   (a) Find the solution of the equation with the initial conditions $y(-\frac{\pi}{2}) = -3$, $y'(-\frac{\pi}{2}) = 4$. Describe the behavior of the solution as $t \to +\infty$.
   (b) Determine λ, $\mu > 0$, $R > 0$ and $\delta \in [0, 2\pi)$ so that the solution obtained in the previous item can be written in the form $e^{\lambda t}R\cos(\mu t - \delta)$ (you can use a calculator to determine an approximate value of δ). Then sketch the graph of this solution.

5. (a) A mass weigh 4 lb stretches a spring 24 in. Assume that there is no damping. If after this the mass is pushed 2 in down and then set in motion with upward velocity of 3 in/s, determine the position u of the mass at any time t. Find the natural frequency, the period, the amplitude, and the phase of the motion (you can use calculator to determine the phase).
   (b) Assume that in the case of the spring-mass system of item (b) there is also a damping and we can change the damping constant. What is the critical damping constant?