Homework Assignment 5 in Differential Equations, MATH308-Spring 2015

due February 13, 2015

Topics covered : *Wronskian, fundamental set of solutions of linear homogeneous equations of second order, linear homogeneous equations of second order with constant coefficients: the case of two distinct real roots of the characteristic polynomial (corresponds to sections 3.2, and 3.1 in the textbook).*

1. (a) Find the Wronskian of the functions $y_1(t) = \cos(\ln t)$, $y_2(t) = \sin(\ln t)$;

   (b) Show that functions $y_1(t)$ and $y_2(t)$ from the previous item are solutions of the differential equation

   $$t^2 y'' + t y' + y = 0, \quad t > 0 \tag{1}$$

   (c) Show that the functions $y_1(t)$ and $y_2(t)$ from item a) constitute the fundamental set of solutions of the equation (1) and find the general solution of the equation (1)

   (d) Find the solution of (1) satisfying the initial conditions $y(1) = 3$, $y'(1) = -5$.

2. Consider differential equations

   $$y'' + 2\alpha y' + (\alpha^2 - 9)y = 0, \tag{2}$$

   where α is a parameter.

   (a) Determine the values of α, if any, for which all solutions tend to zero as $t \to +\infty$.

   (b) In the case $\alpha = -1$ find the solution satisfying the initial condition $y(0) = -3$, $y'(0) = 2$.

3. (**bonus-20 points**) Read the formulation and the proof of Theorem 3.2.6 on the page 153 (Abel's Theorem). Then, using this theorem, find the value of $W(y_1, y_2)(3)$, if the functions $y_1(t)$ and $y_2(t)$ constitute a fundamental set of solutions of the equation $ty'' - 2y' + e^t y = 0$ such that $W(y_1, y_2)(5) = 2$ (for this you do not actually need to find $y_1(t)$ and $y_2(t)$).