Homework Assignment 5 in MATH 308-Spring 2016, Regular section

due March 9 , 2017

Topics covered : *mechanical vibratons (section 3.7);use that the gravitational acceleration* $g = 32\frac{ft}{s^2}$*; Euler equations (section 5.4 pages 272-274), the method of reduction of order (section 3.4); Introduction to systems of first order equations (section 7.1), multiplication of matrices (section 7.2)*

1. (a) A mass weigh 8 lb stretches a spring $\frac{8}{3}$ in. Assume that there is no damping. If after this the mass is pushed 4 in up and then set in motion with downward velocity of 4 in/s, determine the position u of the mass at any time t. Find the natural frequency, the period, the amplitude, and the phase of the motion (you can use calculator to determine the phase).

   (b) Assume that in the case of the spring-mass system of item (b) there is also a damping and we can change the damping constant. What is the critical damping constant?

2. Find the general solution of the following equations:

   (a) $28x^2y'' + 25xy' - 18y = 0, \quad x > 0;$

   (b) $4x^2y'' - 8xy' + 9y = 0, \quad x > 0;$

   (c) $x^2y'' - 13xy' + 113y = 0, \quad x > 0.$

3. It is known that the equation
$$y'' - 4ty' + (4t^2 - 2)y = 0$$
has a solution $y_1(t) = e^{t^2}$. Use the method of reduction of order to find the general solution of this equation.

4. Transform the given equation into a system of first order differential equations:

   (a) $u'' - \arcsin(t^2)u' - 8\cos u = -t^4$

   (b) $y^{(3)} + 12\tan^2(t^3)y' - (2 - 4t)y = 0$

5. Express the given system of linear differential equations in matrix form:

   (a) $$\begin{cases} x_1' &= 4x_1 + 8x_3 \\ x_2' &= 20x_2 - 18x_3 \\ x_3' &= 5x_1 - 13x_2 \end{cases}$$

   (b) $$\begin{cases} x' &= (t^3 - 5t)x + \ln(3t^2 + 1)y - \cos(t^4) \\ y' &= -(t^2 - 3t + 2)\,x + 6\sin(2t)\,y + 20 \end{cases}$$