# Homework Assignment 6 in Differential Equations, MATH308-SUMMER 2012

due to June 13, 2012

Topics covered : *Wronskian, fundamental set of solutions of linear homogeneous equations of second order, linear homogeneous equations of second order with constant coefficients: the case of two distinct real roots of the characteristic polynomial (corresponds to sections 3.2, and 3.1 in the textbook).*

1. (a) Find the Wronskian of the functions $y_1(t) = e^t$, $y_2(t) = t^2 + 2t + 2$;

   (b) Show that functions $y_1(t)$ and $y_2(t)$ from the previous item are solutions of the differential equation

   $$ty'' - (t+2)y' + 2y = 0, \quad t > 0 \tag{1}$$

   (c) Show that the functions $y_1(t)$ and $y_2(t)$ from item a) constitute the fundamental set of solutions of the equation (1).

   (d) Find the solution of (1) satisfying the initial conditions $y(1) = 0$, $y'(1) = 1$.

2. (a) Find the general solution of differential equation

   $$y'' - 3y' - 4y = 0;$$

   (b) Find the solution of the same equation satisfying the initial condition $y(0) = \alpha$, $y'(0) = 1$;

   (c) Find all $\alpha$ for which the corresponding solutions from the previous item approaches zero as $t \to +\infty$;

   (d) Find all $\alpha$ for which the corresponding solutions from item b) approaches zero as $t \to -\infty$.

3. (**bonus-20 points**) Read the formulation and the proof of Theorem 3.2.6 on the page 153 (Abel's Theorem). Then, using this theorem, find the value of $W(y_1, y_2)(4)$, if the functions $y_1(t)$ and $y_2(t)$ constitute a fundamental set of solutions of the equation $ty'' + 3y' + (\cos t)\, y = 0$ such that $W(y_1, y_2)(3) = 1$ (for this you do not actually need to find $y_1(t)$ and $y_2(t)$).