Homework Assignment 7 in Differential Equations, MATH308-Fall 2016

due October 17, 2016

Section covered: 7.3 (finding eigenvalues of matrices), 7.4 (fundamental set of solutions), 7.5 (linear homogeneous systems with constant coefficients : the case of distinct real eigenvalues and n=2)

1. Determine whether the following solutions of the the system $x'(t) = Ax(t)$ form a fundamental set of its solutions. If they do, find general solution of the system.

   (a) $x^1 = \begin{pmatrix} -4e^{-5t} \\ 3e^{-5t} \\ 2e^{-5t} \end{pmatrix}$, $\quad x^2 = \begin{pmatrix} e^{3t}\cos 2t \\ -e^{3t}\sin 2t \\ -2e^{3t}\sin 2t \end{pmatrix}$, $\quad x^3 = \begin{pmatrix} e^{3t}\sin 2t \\ e^{3t}\cos 2t \\ 2e^{3t}\cos 2t \end{pmatrix}$

   (b) $x^1 = e^{-7t}\begin{pmatrix} 10 \\ -14 \end{pmatrix}$, $\quad x^2 = e^{-7t}\begin{pmatrix} -5 \\ 7 \end{pmatrix}$

2. Given the following system of linear differential equations:

$$\begin{cases} x_1' &= -8x_1 - 10x_2 \\ x_2' &= 5x_1 + 7x_2 \end{cases} \tag{1}$$

   (a) Find the general solution of the system (1).

   (b) Find the solution of the the system (1) satisfying the initial conditions: $x_1(0) = 3, \quad x_2(0) = -2$.

   (c) Find all α_1 and α_2 such that if $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ is the solution of of the system (1) with initial condition $x(0) = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$ then $x(t) \to 0$ as $t \to \infty$.

   (d) Find all β_1 and β_2 such that if $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ is the solution of of the system (1) with initial condition $x(0) = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$ then $x(t) \to 0$ as $t \to -\infty$.

3. Find the **eigenvalues** of the following matrix

$$A = \begin{pmatrix} -2 & 3 & -1 \\ 4 & -1 & 1 \\ -4 & -3 & -5 \end{pmatrix}.$$

   It is known that A has at least one integer eigenvalue.