1. Given the solution $y_{1}(t)=t^{-1 / 2}$ of the differential equation $4 t^{2} y^{\prime \prime}-4 t y^{\prime}-5 y=0, \quad t>0$. Use the method of reduction of order to find a second solution $y_{2}(t)$ of this equation such that $\left\{y_{1}(t), y_{2}(t)\right\}$ is a fundamental set of solutions on $t>0$.
2. (a) For each of the following equations write down the form in which a particular solution should be found according to the method of undetermined coefficients (you do not need to find the value of the undetermined coefficient/coefficients here):
i) $3 y^{\prime \prime}+5 y^{\prime}-2 y=7 e^{2 t}$;
ii) $3 y^{\prime \prime}+5 y^{\prime}-2 y=7 e^{-2 t}$
iii) $3 y^{\prime \prime}+5 y^{\prime}-2 y=3 e^{2 t} \cos 10 t$;
iv) $3 y^{\prime \prime}+5 y^{\prime}-2 y=7 e^{-2 t} \cos 3 t$;
v) $4 y^{\prime \prime}-4 y^{\prime}+y=5 e^{3 t}$;
vi) $4 y^{\prime \prime}-4 y^{\prime}+y=5 e^{t / 2}-2 e^{t / 2} \sin 3 t$;
vii) $y^{\prime \prime}+\omega_{0}^{2} y=\cos \omega t+2 \sin \omega t$ (consider separately the case $\omega^{2} \neq \omega_{0}^{2}$ and the case $\left.\omega^{2}=\omega_{0}^{2}\right) ;$
viii) $18 y^{\prime \prime}+30 y^{\prime}+17 y=e^{-5 t / 6}\left(\cos \left(\frac{t}{4}\right)-2 \sin \left(\frac{t}{4}\right)\right)$.
(b) Find the general solution for equation in the item (a) iii);
(c) Find the general solution for equation in the item (a) vii) (consider separately the case $\omega^{2} \neq \omega_{0}^{2}$ and the case $\omega^{2}=\omega_{0}^{2}$ ).
